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Dynamical Behaviour of Three Competing Plant Species through Mathematical Modeling

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Abstract:

In this paper, a three species non – linear Lotka –Voltera competition model has been proposed. Different equilibrium points are determined. Around these equilibrium points, stability of the system has been analysed. Finally, numerical simulation results have been presented to study the dynamical behavior of proposed model.

Keywords: *Nonlinear competition model, Equilibrium points, stability Analysis, Hopf Bifurcation*.

1. Introduction

Mathematical models can be used in the study of ecological system. Mathematical model is one of the most important research topics in ecological system, since the pioneering work of Lotka (1) and Voltera (2). It is well known fact that the mathematical models can be solved the complicated dynamics of ecological system. In earlier, two species interaction ecological models have been focused in most of the research. May (3) and Hassel (4). Then three species ecological model have been proposed and their complexity have been reported (5, 6, 7, 8, 9, 10, 11, 12, 13 14, 15).

2. Formulation of Mathematical Model

We consider three species Lotka - Voltera competition model

With logistic growth. The intra – specific and inter – specific competition between three plant species competing for limited sources like sunlight , water , soil , land and nutrients. This is described by following ordinary differential equations.

$$\frac{dX}{dt} = r_1X\left(1 - \frac{X}{k1} - a_{12}\frac{Y}{k1} - a_{13}\frac{Z}{k1}\right)$$

$$\frac{dY}{dt} = r_2Y\left(1 - \frac{Y}{k2} - a_{21}\frac{X}{k2} - a_{23}\frac{Z}{k2}\right.\ \ldots\ldots\ldots \qquad (1)$$

$$\frac{dZ}{dt} = r_3Z\left(1 - \frac{Z}{k3} - a_{31}\frac{X}{k3} - a_{32}\frac{Y}{k3}\right)$$

Where X(t) , Y (t) , Z (t) are densities of three different species at time t . r_1, r_2 and r_3 are intrinsic growth rates , k_1 , k_2 , k_3 are carrying capacities of species X , Y and Z respectively , a_{12} , a_{13} measures competitive effect of Y and Z on X , a_{21} , a_{23} measures competitive effect of X and Z on Y and a_{31} , a_{32} measures competitive effect of X and Y on Z respectively .

3. Equilibrium Points

An equilibrium point exists if

$$\frac{dX}{dt} = F_1(X, Y, Z) = 0$$

$$\frac{dY}{dt} = F_2(X, Y, Z) = 0$$

$$\frac{dZ}{dt} = F_3(X, Y, Z) = 0$$

Using Non – dimensional Quantities

$N_1 = \frac{X}{k1}$ $\alpha_{12} = a_{12}\frac{k2}{k1}$ $\alpha_{13} = a_{13}\frac{k3}{k1}$

$N_2 = \frac{Z}{k3}$ $\alpha_{21} = a_{31}\frac{k1}{k3}$ $\alpha_{13} = a_{32}\frac{k2}{k3}$

$\rho = \frac{r2}{r1}$ $\rho' = \frac{r3}{r1}$ $t' = r_1 t$

We obtain

$$\frac{dN1}{dt} = N_1 (1 – N_1 - \alpha_{12}N_2 - \alpha_{13}N_3) = 0 = f_1 (N_1 , N_2 , N_3)$$

$$\frac{dN2}{dt} = \rho N_2 (1 – N_2 - \alpha_{21}N_1 - \alpha_{23}N_3) = 0 = f_2 (N_1 , N_2 , N_3)$$

$$\frac{dN3}{dt} = \rho' N_3 (1–N_3-\alpha_{31}N_1-\alpha_{32}N_2) = 0 = f_3 (N_1, N_2, N_3) \quad \ldots\ldots\ldots(2)$$

Let (N_1^*, N_2^* , N_3^*) be any arbitrary equilibrium point of system(2).

The system(2) have following possible equilibrium points

- The trivial equilibrium point $E_0 = (0 , 0 , 0)$
- The axial equilibrium point $E_1 = (1, 0, 0)$
- The axial equilibrium point $E_3 = (0 , 0 , 1)$
- The boundary equilibrium point $E_4 = (\frac{1-\alpha12}{1-\alpha12\alpha21} , \frac{1-\alpha21}{1-\alpha12\alpha21} , 0)$ if $\alpha_{12} < 1$, $\alpha_{21} < 1$ and $\alpha_{12}\alpha_{21} < 1$
- The boundary equilibrium point $E_5 = (\frac{1-\alpha13}{1-\alpha13\alpha31} , 0 , \frac{1-\alpha31}{1-\alpha13\alpha31})$ if $\alpha_{13} < 1$, $\alpha_{31} < 1$ and $\alpha_{13}\alpha_{31} < 1$
- Another boundary equilibrium point $E_6 = (0 , \frac{1-\alpha23}{1-\alpha23\alpha32} , \frac{1-\alpha32}{1-\alpha23\alpha32})$ if $\alpha_{23} < 1$, $\alpha_{32} < 1$ and $\alpha_{23}\alpha_{32} < 1$
- The coexistence equilibrium point $E_* = (x^* , y^* , z^*)$ where $x^* = \frac{1-\alpha12-\alpha13+\alpha12\alpha23+\alpha13\alpha32-\alpha23\alpha32}{Dr}$

$$Y^* = \frac{1-\alpha21-\alpha23+\alpha21\alpha13+\alpha23\alpha31-\alpha31\alpha13}{Dr} \qquad z^* = \frac{1-\alpha31-\alpha32+\alpha31\alpha12+\alpha32\alpha21-\alpha12\alpha21}{Dr}$$

Where $Dr = \alpha_{12}\alpha_{23}\alpha_{31} + \alpha_{13}\alpha_{32}\alpha_{21} - \alpha_{12}\alpha_{21} - \alpha_{23}\alpha_{32} - \alpha_{31}\alpha_{13} > 0$

If x^* exists if $1 + \alpha_{12}\alpha_{23} + \alpha_{13}\alpha_{32} > \alpha_{12} + \alpha_{13} + \alpha_{23}\alpha_{32}$

Y^* exists if $1 + \alpha_{21}\alpha_{13} + \alpha_{23}\alpha_{31} > \alpha_{21} + \alpha_{23} + \alpha_{31}\alpha_{13}$

Z^* exists if $1 + \alpha_{31}\alpha_{12} + \alpha_{32}\alpha_{21} > \alpha_{31} + \alpha_{32} + \alpha_{12}\alpha_{21}$

4. STABILITY ANALYSIS

We first determine Jacobian matrix

$$J = \begin{pmatrix} 1-2N_1^* - \alpha_{12}N_2^* - \alpha_{13}N_3^* & -\alpha_{12}N_1^* & -\alpha_{13}N_1^* \\ -\rho\alpha_{21}N_2^* & \rho(1-2N_2^* - \alpha_{21}N_1^* - \alpha_{23}N_3^*) & -\rho\alpha_{23}N_2^* \\ -\rho'\alpha_{31}N_3^* & -\rho'\alpha_{32}N_3^* & \rho'(1-2N_3^* - \alpha_{31}N_1^* - \alpha_{32}N_2^* \end{pmatrix}$$

Claim 1. The equilibrium point $E_0 = (0,0,0)$ is a saddle point .

Proof. The characteristic equation of Jacobian matrix J at E_0 is given by

$$(\lambda - 1) (\lambda - \rho) (\lambda - \rho') = 0$$

So the roots of the above equation are $1 , \rho , \rho'$. Since all parameters related to this mathematical model are positive, hence the trivial equilibrium point is a saddle point.

Claim 2. The equilibrium point $E_1 = (1 , 0 , 0)$ is locally asymptotically stable if $\alpha_{21} > 0$ and $\alpha_{31} > 0$.

Proof. The characteristic equation of Jacobian matrix J at the point E_1 is given by

$$(\lambda + 1)\ (\lambda + \rho\alpha_{21})\ (\lambda + \rho'\alpha_{31}) = 0$$

The roots of the above equation are -1, $-\rho\alpha_{21}$, $-\rho'\alpha_{31}$. The equilibrium point E_1 is locally asymptotically stable if $\alpha_{21} > 0$ and $\alpha_{31} > 0$.

Claim 3. The equilibrium point $E_2 = (0, 1, 0)$ is locally asymptotically stable if $1 < \alpha_{12}$ and $1 < \alpha_{32}$

Proof. The characteristic equation at E_2 is given by

$$[\lambda - (1-\alpha_{12})]\ (\lambda + \rho)\ [\lambda - \rho'(1 - \alpha_{32})] = 0$$

The roots of the above equation are $-\rho$, $(1 - \alpha_{21})$, $\rho'(1 - \alpha_{32})$. Therefore the equilibrium point E_2 is locally asymptotically stable if $1 < \alpha_{21}$ and $1 < \alpha_{32}$.

Claim 4. The equilibrium point $E_3 = (0, 0, 1)$ is locally asymptotically stable if $1 < \alpha_{13}$ and $1 < \alpha_{23}$.

Proof. The characteristic equation at E_3 is given by

$$[\lambda - (1-\alpha_{13})]\ [(\lambda - \rho\,(1-\alpha_{23})]\ (\lambda + \rho')] = 0$$

The roots of the above equation are $-\rho'$, $(1 - \alpha_{13})$, $\rho(1 - \alpha_{23})$. Therefore the equilibrium point E_3 is locally asymptotically stable if $1 < \alpha_{13}$ and $1 < \alpha_{23}$.

Claim 5. The equilibrium point E_4 is locally asymptotically stable.

Proof. The characteristic equation of J at $E_4 = (x_1, y_1, 0)$ where $x_1 = \frac{1-\alpha_{12}}{1-\alpha_{12}\alpha_{21}}$, $y_1 = \frac{1-\alpha_{21}}{1-\alpha_{12}\alpha_{21}}$, E_4 exists if $\alpha_{12} < 1$, $\alpha_{21} < 1$ and $\alpha_{21}\alpha_{12} < 1$ is $[\rho'(1 - \alpha_{31}x_1 - \alpha_{32}y_1) - \lambda]\ (\lambda^2 + A_1\lambda + A_2) = 0$ where

$$A_1 = -\rho - 1 + (2 + \alpha_{21})\, x_1 + (2 + \alpha_{12})\, y_1 \text{ and } A_2 = \rho\,[1 - 2(x_1 + y_1) - \alpha_{21}(x_1 - 2x_1^2) - \alpha_{12}(y_1 - 2y_1^2) + 4x_1y_1]$$

One of the root of above characteristic equation is $\rho'(1 - \alpha_{31}x_1 - \alpha_{32}y_1)$ and other two roots can be obtained from the equation $\lambda^2 + A_1\lambda + A_2 = 0$. By Ruth – Hurwitz criteria, the roots of the equation are negative real number or negative real part of complex roots if $A_1 > 0$, $A_2 > 0$.

(ie) $\rho < 2(x_1+y_1) + \alpha_{21}x_1 + \alpha_{12}y_1 - 1$ and $\alpha_{21} < \frac{1-2x_1-2y_1-\alpha_{12}y_1+2\alpha_{12}y_1^2+4x_1y_1}{x_1-2x_1^2}$

So the equilibrium point is locally asymptotically stable if $1 < \alpha_{31}x_1 + \alpha_{32}y_1$, $\rho < 2(x_1+y_1) + \alpha_{21}x_1 + \alpha_{12}y_1 - 1$ and $\alpha_{21} < \frac{1-2x_1-2y_1-\alpha_{12}y_1+2\alpha_{12}y_1^2+4x_1y_1}{x_1-2x_1^2}$

Claim 6. The equilibrium point E_5 is locally asymptotically stable.

Proof. The characteristic equation of J at $E_5 = (x_2, 0, z_2)$ where $x_2 = \frac{1-\alpha_{13}}{1-\alpha_{13}\alpha_{31}}$, $z_2 = \frac{1-\alpha_{31}}{1-\alpha_{13}\alpha_{31}}$. The equilibrium point E_5 exists if $\alpha_{13} < 1$, $\alpha_{31} < 1$ and $\alpha_{31}\alpha_{13} < 1$ is $[\rho(1 - \alpha_{21}x_2 - \alpha_{23}z_2) - \lambda]\ (\lambda^2 + B_1\lambda + B_2) = 0$ where $B_1 = -\rho' - 1 + 2(x_2 + z_2) + \alpha_{31}x_2 + \alpha_{13}z_2$ and $B_2 = \rho'\,[1 - 2(x_2 + z_2) + \alpha_{31}(2x_2^2 - x_2) + \alpha_{13}(2z_2^2 - z_2) + 4x_2y_2]$.

One of the root of above characteristic equation is $\rho(1 - \alpha_{21}x_2 - \alpha_{23}z_2)$ and other two roots can be obtained from the equation $\lambda^2 + B_1\lambda + B_2 = 0$. By Ruth – Hurwitz criteria, the roots of the equation are negative real number or negative real part of complex roots if $B_1 > 0$, $B_2 > 0$.

So the equilibrium point E_5 is locally asymptotically stable if $1 < \alpha_{21}x_2 + \alpha_{23}z_2$, $\rho' < 2(x_2+z_2) + \alpha_{31}x_2 + \alpha_{13}z_2 - 1$ and $\alpha_{31} < \frac{1-2(x_2+z_2)+\alpha_{13}(2z_2^2-z_2)+4x_2z_2}{x_2-2x_2^2}$.

Claim 7. The equilibrium point E_6 is locally asymptotically stable

Proof. The characteristic equation of Jacobian J at the point $E_4 = (0, y_3, z_3)$ where $y_3 = \frac{1-\alpha_{23}}{1-\alpha_{23}\alpha_{32}}$, $z_3 = \frac{1-\alpha_{32}}{1-\alpha_{23}\alpha_{32}}$, E_6 exists if $\alpha_{23} < 1$, $\alpha_{32} < 1$ and $\alpha_{23}\alpha_{32} < 1$ is $[(1 - \alpha_{12}y_3 - \alpha_{13}z_3) - \lambda]\ (\lambda^2 + C_1\lambda + C) = 0$ where

$C_1 = -\rho\rho'[2 - 2(y_3 + z_3) - \alpha_{32}y_3 - \alpha_{23}z_3]$ and $C_2 = \rho\rho'[1 - 2(y_3 + z_3) + \alpha_{32}(y_3 - 2y_3^2) - \alpha_{23}(z_3 - 2z_3^2) + 4y_3z_3]$

One of the root of above characteristic equation is $\rho'(1-\alpha_{31}x_1-\alpha_{32}y_1)$ and other two roots can be obtained from the equation $\lambda^2 + A_1\lambda + A_2 = 0$. By Ruth – Hurwitz criteria, the roots of the equation are negative real number or negative real part of complex roots if $A_1 > 0$, $A_2 > 0$.

(ie) $\rho < 2(x_1+y_1) + \alpha_{21}x_1 + \alpha_{12}y_1 - 1$ and $\alpha_{21} < \frac{1-2x_1-2y_1-\alpha_{12}y_1+2\alpha_{12}y_1^2+4x_1y_1}{x_1-2x_1^2}$

Claim 8. The Equilibrium point $E_*=(x^*,y^*,z^*)$ is locally asymptotically stable.

Proof. The characteristic equation of Jacobian matrix J at E_* is given by $\lambda^3 + \Psi_1\lambda^2 + \Psi_2\lambda + \Psi_3 = 0$ ……..(3) where $M_{11} = 1-2x^* - \alpha_{12}y^* - \alpha_{13}z^*$, $M_{12} = -\alpha_{12}x^*$, $M_{13} = -\alpha_{13}x^*$, $M_{21} = -\rho\alpha_{21}y^*$, $M_{22} = \rho(1-2y^*-\alpha_{21}x^*-\alpha_{23}z^*)$, $M_{23} = -\rho\alpha_{23}y^*$, $M_{31} = -\rho'\alpha_{31}z^*$, $M_{32} = -\rho'\alpha_{32}z^*$, $M_{33} = \rho'(1-2z^*-\alpha_{31}x^*-\alpha_{32}y^*)$ and $\Psi_1 = -(\text{trace } J) = -M_{11} - M_{22} - M_{33} = (2+\rho\alpha_{21}+\rho'\alpha_{31})x_* + (\alpha_{12}+2\rho+\rho'\alpha_{32})y^* + (\alpha_{13}+\rho\alpha_{23}+2\rho')z^* - (1+\rho+\rho')$, Ψ_2 = sum of minor of principal diagonal elements of J = $M_{11}M_{22} + M_{22}M_{33} + M_{33}M_{11} - M_{12}M_{21} - M_{23}M_{32} - M_{31}M_{13}$ and $\Psi_3 = -\det(J) = M_{11}M_{22}M_{33}+M_{12}M_{23}M_{31}+M_{13}M_{32}M_{21}-M_{23}M_{32}M_{11}-M_{12}M_{21}M_{33}-M_{13}M_{31}M_{22}$

According to Ruth-Hurwitz criteria, the above characteristic equation have negative real root or pair of imaginary roots with negative real part if $\Psi_1>0$, $\Psi_3>0$ and $\Psi_1\Psi_2>\Psi_3$ holds.

5. Hopf Bifurcation

For the variation of different parameters, different dynamical behavior occur in the system .We choose a competitive coefficient α_{12} as the bifurcation parameter. The conditions for existence of Hopf bifurcation at $\alpha_{12} = \alpha_{12}*$ are stated as follows.

- $\Psi_i(\alpha_{12}*) > 0$, i = 1,2,3
- $\Psi_1(\alpha_{12}*)\,\Psi_2(\alpha_{12}*) - \Psi_3(\alpha_{12}*) = 0$
- Re $\left(\frac{d\lambda_i}{d\alpha_{12}}\right)$ at $\alpha_{12} = \alpha_{12}* \neq 0$ i = 1, 2, 3 where λ_i are roots of the equation.

The characteristic equation at equilibrium point E_* is given by

$\lambda^3 + \Psi_1\lambda^2 + \Psi_2\lambda + \Psi_3 = 0$ …..(3).

When the bifurcation parameter reaches the critical value $\alpha_{12} = \alpha_{12}*$, (3) becomes $(\lambda^2 + \Psi_2)(\lambda + \Psi_1) = 0$ its roots are $\lambda_1 = i\sqrt{\Psi 2}$ are $\lambda_2 = -i\sqrt{\Psi 2}$ and $\lambda_3 = -\Psi_1$.

Let us assume that for $\alpha_{12} \in (\alpha_{12}^*-\epsilon, \alpha_{12}^*+\epsilon)$, the roots of the character equation are $\lambda_1(\alpha_{12}) = \delta_1(\alpha_{12}) + i\delta_2(\alpha_{12})$, $\lambda_2(\alpha_{12}) = \delta_1(\alpha_{12}) - i\delta_2(\alpha_{12})$, $\lambda_3\alpha_{12}) = -\Psi_1$.

For transversality condition Re $\left(\frac{d\lambda_i}{d\alpha_{12}}\right)$ at $\alpha_{12} = \alpha_{12}* \neq 0$, i = 1, 2, 3

Substituting $\lambda_1(\alpha_{12}) = \delta_1(\alpha_{12}) + i\delta_2(\alpha_{12})$ in (3) and differentiating we get $A(\alpha_{12})\,\delta_1'(\alpha_{12}) - B(\alpha_{12})\,\delta_2(\alpha_{12}) + P(\alpha_{12}) = 0$, $B(\alpha_{12})\,\delta_1'(\alpha_{12}) + A(\alpha_{12})\,\delta_2(\alpha_{12}) + Q(\alpha_{12}) = 0$

The transversality condition Re $\left(\frac{d\lambda_i}{d\alpha_{12}}\right)$ at $\alpha_{12} = \alpha_{12}* \neq 0$, i = 1, 2, 3

And $A(\alpha_{12}*) = -2\Psi_2(\alpha_{12}*)$, $B(\alpha_{12}*) = 2\Psi_1(\alpha_{12}*)\sqrt{\Psi_2(\alpha_{12}*)}$, $P(\alpha_{12}*) = \Psi_3'(\alpha_{12}*) - \Psi_2(\alpha_{12}*)\Psi_1'(\alpha_{12}*)$, $Q(\alpha_{12}*) = -\Psi_2'(\alpha_{12}*)\sqrt{\Psi 2(\alpha_{12}*)}$. Therefore

$\left[\text{Re}\left(\frac{d\lambda_i}{d\alpha_{12}}\right)\right]$ at $\alpha_{12} = \alpha_{12}* = -\frac{AP+BQ}{A^2+B^2} \neq 0$ for i = 1,2 if $AP + BQ \neq 0$ and $\lambda_3(\alpha_{12}^*) = \Psi_1 \neq 0$. Therefore transversally conditions are satisfied and Hopf bifurcation occurs at $\alpha_{12} = \alpha_{12}^*$.

6. Numerical Simulation

We have solved system (2) numerically by MATLAB. A set of parametric values have been collected from Department of Botany, St. Andrews College, Gorakhpur. $r_1 = 3.8$, $r_2 = 2.5$, $r_3 = 0.4$, $K_1 = 175$, $K_2 = 75$, $K_3 = 50$, $\alpha_{12} = 0.2151$, $\alpha_{23} = 0.336$ and $\alpha_{31} = 1.75$, $\rho = 0.6578$, $\rho' = 0.1052$ If we take $\alpha_{12} = \alpha_{32}$, $\alpha_{23} = \alpha_{13}$, $\alpha_{31} = \alpha_{21}$, the equilibrium point is (0.6516,0.5687,0.6723) . At this point, Ruth's-Hurwitz criteria is satisfied. Also the eigenvalues of the Jacobian matrix at this equilibrium point are -0.7148, -

0.3215, -0.0597. Therefore the conditions of local stability are satisfied. It shows in fig1(a).We plot phase portrait diagram of the system(2) in fig1(b) by taking same set of parametric values used in fig1(a).Hence it can be concluded that the equilibrium point (0.6516,0.5687,0.6723) is locally asymptotically stable for this set of parameters.

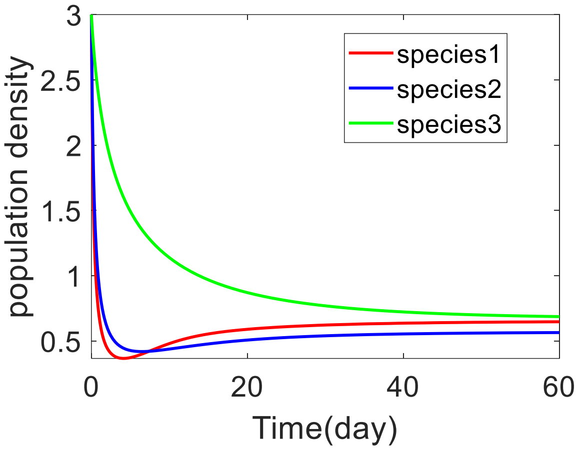

Fig 1(a)

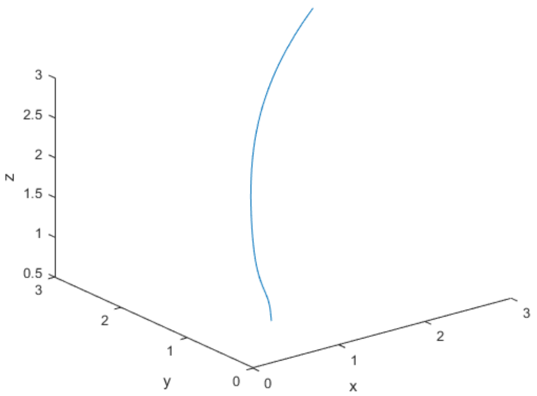

Fig 1(b)

Fig 2(a) and Fig 2(b-d) shows oscillatory behavior and bifurcation diagram of the system (2) with respect to $\alpha_{12} = 1.5$ have been drawn by taking parametric values $\alpha_{23} = 0.336$, $\alpha_{31} = 0.315$, $\rho = 0.6578, \rho' = 0.1052$. From fig 2(b-d), for Hopf bifurcation, using theorem we have calculated the critical value of α_{12} is 1.2899 .Also the system shows oscillatory behavior in ($1.2899< \alpha_{12}<1.3096$) from Fig 2(b-d). Bifurcation diagram of the system (2) of x, y,z species shows stable steady state and extinction in ($0.1< \alpha_{12}<1.2899$) and ($1.30906\leq\alpha_{12}<2$) respectively with respect to α_{12}.

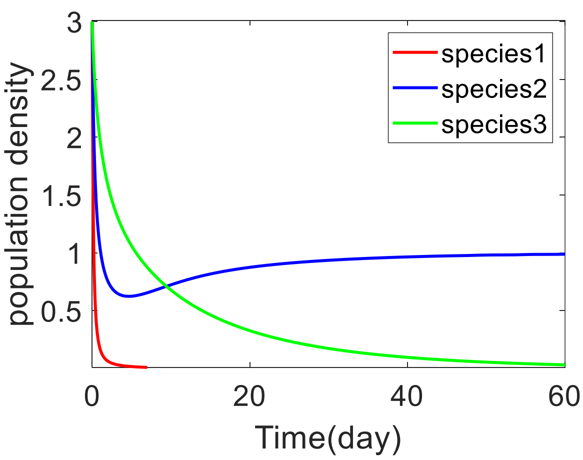

Fig 2(a)

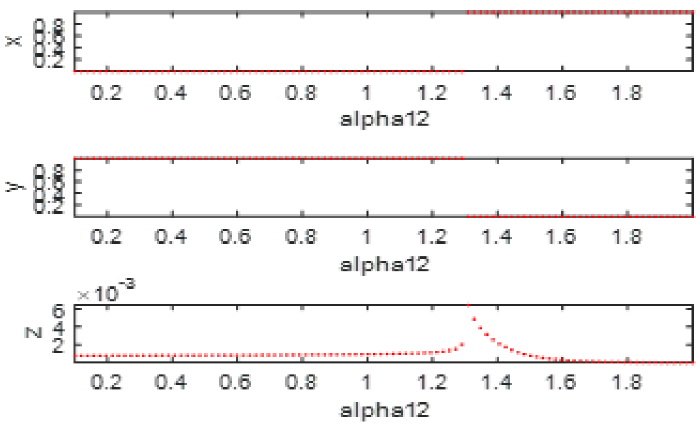

Fig 2(b-d)

Using parametric values α_{12} =0.2151, α_{31} = 0.315, ρ =0.6578, ρ' = 0.1052, the existence of oscillatory behavior and bifurcation diagram of the system (2) with respect to α_{23} =1.226 have been shown in Fig. 3 (a) and 3 (b-d) respectively. The critical value of α_{23} is 0.6373.The system shows oscillatory behavior in (0.6373<α_{23}<1.5556). Extinction and Stable steady state in (0.1<α_{23}<0.6373) and (1.5556<α_{23}<2) for x species. For y species, only extinction in (0.1<α_{23}<0.6373) and (1.5556<α_{23}<2). Stable steady state and extinction in (0.1 <α_{23}<0.6373) and (1.5556<α_{23}<2) for z species.

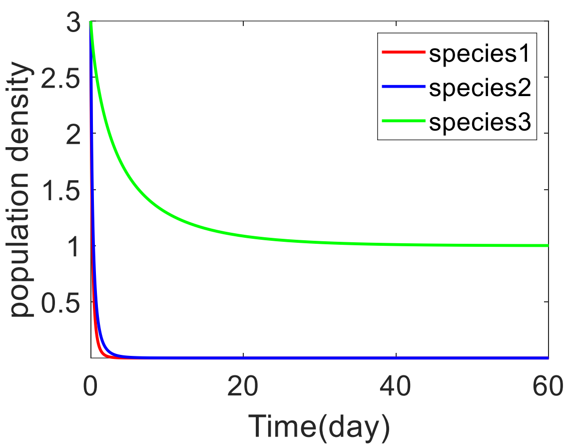

Fig 3(a)

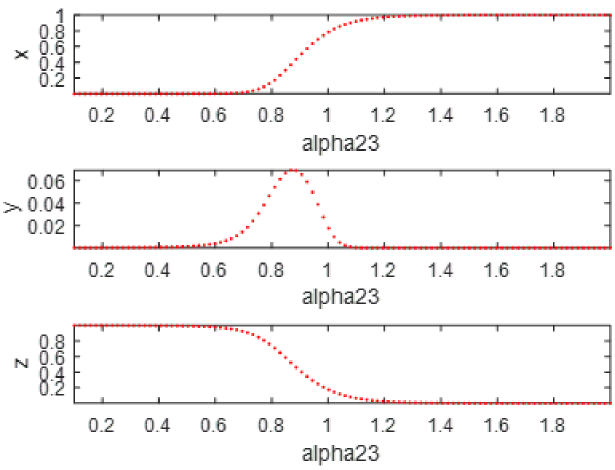

Fig 3(b-d)

From fig4(a) and 4(b-d) shows existence of oscillatory behavior and bifurcation of the system(2) using the parametric values α_{12} =0.2151, α_{23} = 0.226, ρ = 0.6578, ρ' =0.1052, with respect to α_{31}=1.75, we have calculated the critical value of α_{31} as 0.1.From Fig 4(b-d) system shows existence of oscillatory behavior in ($0.1<\alpha_{31}<1.02121$). Also, the system (2) shows stable steady state for x species and extinction for y and z species in ($1,02121< \alpha_{31}<2$) .

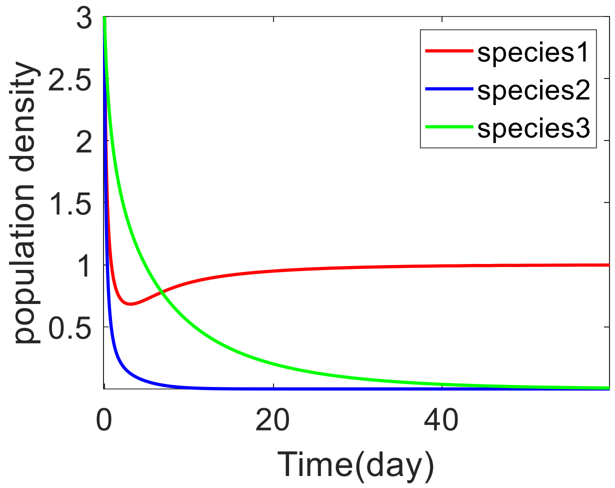

Fig 4(a)

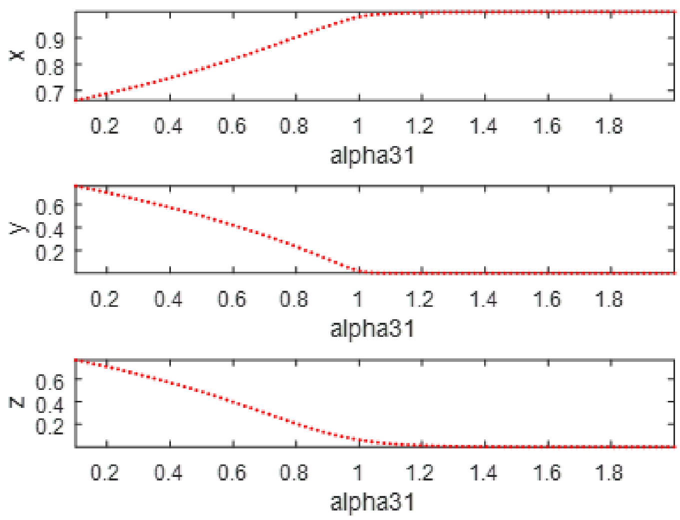

Fig 4(b-d)

7. Conclusion

This paper deals with non-linear logistic growth of three plant species and the intra and inter specific competition model has been formulated. Equilibrium points have been calculated and their stability conditions have also been analysed. It is observed that the inter species competition coefficients of the species have significant role in the stability of an ecological system. Species extinction is possible depending on the competition coefficients.

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