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The Content, Methods, and Significance of "Intelligent Fatigue Statistics"

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Abstract: *The main purpose of this paper is to introduce the past life of "Intelligent Fatigue Statistics", how it came into being, how it has evolved and its future. Focusing on the significance of the three unique views of Mr. Zhentong Gao, an academician of the Chinese Academy of Sciences, on the three-parameters Weibull distribution, especially the physical significance of the positional parameter of the Weibull distribution that he proposed is the safety life with 100% reliability, which is exactly the theoretical basis of Zhentong Gao method. The Zhentong Gao method is the important method of fatigue statistics intelligence. With the promotion and enhancement of the Zhentong Gaomethod, it further proves that the fatigue statistics intelligence is not only feasible but also necessary.*

Keywords: *Intelligent Fatigue Statistics, Three-parameters Weibull distribution, Estimation of the Three parameters of the Weibull distribution, Zhentong Gao method.*

1. Introduction

For those who are engaged in structural fatigue, reliability and other related aspects of the work or research Weibull distribution will not feel strange, but will also feel some kind of helplessness in the estimation of its three parameters, graphical method (Gao, 1986) is too rough and inconvenient, the so-called analytical method (Gao and Xiong, 2000) to solve the transcendental system of equations is a very cumbersome thing, and there will be a Weibull distribution of the location of the parameter is greater than that of the fatigue life given the minimum value of such problems (Xu, 2021). Of course, there are also the least squares method (LSM) (Fu and Gao,1990), maximum likelihood method (MLE) (Yan et al,2001; Hu et al,2000), and so on. However, no matter which method is used, the mathematical derivation is complicated and the calculation is not easy. More importantly, the calculation results are not ideal. Then how to solve these problems? In the past, there are two ways: one is to reduce the three parameters of the Weibull distribution by one, i.e., its position parameter to zero (Zhang et al, 2010), so there are only two parameters left (shape parameter and scale parameter) makes the complexity of the problem greatly reduced; the other way is to take the logarithm of the asymmetric original data (Gao, 1986)) so that the data taken by the logarithm of the data is closer to the symmetry of the data and then fit it with a Gaussian distribution, namely, simply Weibull distribution is not used. Mr. Gao believes that this is not the solution to the problem, and he has been thinking about and proposed the characteristics of the Weibull distribution for decades.

1. The CDF and PDF of the Weibull distribution are, respectively,

$$F(x)=1-\exp\{-[(x-x_0)/\lambda]^b\} \quad (1)$$

$$f(x)=(b/\lambda)\,[(x-x_0)/\lambda)]^{b-1}\exp\{-[(x-x_0)/\lambda]^b\} \quad (2)$$

It follows that its position parameter x_0 must satisfy $x_0 < x$; and it is physically significant in the field of structural fatigue life with 100% reliability (Xu, 2021; Gao and Xu, 2022). where b and λ are the shape and scale parameters of the Weibull distribution, respectively.

2. The Weibull distribution is a full-state distribution that can be fitted to left- or right-partial data, as well as symmetric data (Xu, 2021; Gao and Xu, 2022).

3. Taking the logarithm of the fatigue life data is mathematically unproblematic, but distorts the physical meaning of the data, with the result that data that should have been fitted with a Weibull distribution becomes fitted with a Gaussian distribution. This is not appropriate (Xu and Gao, 2023; Gao and Xu, 2025). And these three points became the theoretical basis of Intelligent Fatigue Statistics. Based on this foundation, the authors proposed the Zhentong Gao method (Xu, 2021; Gao and Xu. 2022; Gao and Xu,2025) by utilizing the characteristics of computers, which made the first step of "Intelligent" in Fatigue Statistics. In this paper, we would like to make a brief review of the history of "Intelligent Fatigue Statistics" from its birth to its development for the readers who are interested in it, so, as to help them to understand its contents, methods and meanings in a deeper way.

2. Origin of the Chinese and English versions of "Intelligent Fatigue Statistics"

In a sense, the birth of the Chinese (Gao and Xu(2022)) and English (Gao and Xu(2025)) editions of Intelligent Fatigue Statistics was somewhat dramatic. In 1963, I enrolled in the Department Mathematics and Mechanics of Beihang University and became a student of Mr. Gao. To repay Mr. Gao for the kindness of his acquaintance (can be referred to https://news.sciencenet.cn/htmlnews/2023/5/499917.shtm), I went to Beijing to meet Mr. Gao at the end of 2019, with the original intention of republishing Mr. Gao's old work "Applied Statistics of Fatigue" (Gao, 1986)) or "Fatigue Reliability" (Gao and Xiong, 2000) to revise some technical issues as well as to add some new algorithms, especially those in machine learning, in fatigue statistics. However, Mr. Gao disagreed and wanted to start a new book with the title "Intelligent Fatigue Statistics". He also asked for 2-3 papers to be published before publishing the book, making the gist of the book available to the public so that he could hear from the peers. Under the direct guidance of Mr. Gao. This task was accomplished, The three papers are " Zhentong Gao Method in the intelligentization of Statistics in Fatigue " (Xu, 2021) "Further Research on Fatigue Statistics Intelligence"(Xu and Gao, 2022)and "Digital Experiment for Estimating Three Parameters and Their Confidence Intervals of Weibull Distribution" (Xu, 2022) published in (Journal of Beihang University) (Acta Aeronautica et Astronautical Sinica) and (International Journal of Science, Technology and Society) respectively in 2021 to 2022. Third paper introduced the Bootstrap in machine learning (Efron and Hastie,2019). It is these three papers that form the core of the Chinese edition of "Intelligent Fatigue Statistics", and Chapter 10 of the book is formed almost in accordance with the third paper.

After the Chinese edition of "Intelligent Fatigue Statistics" was published in October 2022 in Beijing, Mr. Gao thought that the unique insights about the Weibull distribution and the method of intelligently determining the three parameters of the Weibull could and should be shared with foreign counterparts, and suggested an English edition. Neither Mr. Gao nor I had any experience in publishing books with foreign publishers, so I randomly found one of the top 10 publishers in the world. I sent an email to CRC Press telling them our intention to publish an English version of "Intelligent Fatigue Statistics", and they offered to send a chapter of the book to them for review, so I sent them the most representative chapter, Chapter 8, " Zhentong Gao method ". Almost 3 months later (2023-10-17), they emailed me a detailed CRC Review Report.

The two anonymous reviewers invited by CRC Publishing were divided into "Worldwide Sales Potential", "Table of Contents", "Author's Suggested Publication Schedule", "Writing style and language", "Possible aids", "Strengths", "Weaknesses", "Suggestions", and "Recommended Books" are nine aspects of their opinions, which are very specific and pertinent. The two reviewers are confident that the book will sell well worldwide, "The application areas include major fields such as aerospace, automotive, and electronics. The quantitative methods and computer-based tools discussed are in line with global technical capabilities and needs." "The combination of these areas of fatigue, probabilistic modeling, and artificial intelligence ...... makes this book very popular and will be a hot topic of interest in the coming years. Extra attention will be paid to the book's breakthroughs in addressing the fundamental concepts of the programming languages used and the traditional aspects of probabilistic modeling. Comparing these traditional approaches with the innovative ones proposed by the authors makes this book will be an even more remarkable piece of literature." "This book takes full advantage of the authors' expertise to provide comprehensive and up-to-date coverage of topics that not only emphasizes practical usage and applications, but also offers a unique perspective on fatigue statistics:(1) Practicality: emphasizes real-world applications and problem solving using computational tools such as Python; (2) Technical Depth: suitable for graduate students and practicing engineers in terms of mathematical and statistical technical depth: mathematical and statistical rigor for graduate students and practicing engineers; (3) cutting-edge research and perspectives: incorporating innovative techniques such as Zhentong Gao method; and timeliness and uniqueness: meeting the modern needs of fatigue problems using the latest computational tools". They "agreed to continue the program of publishing the book with full confidence in it. The best indicator of the book's success is the logical structure of its content and knowledge of computer science, statistics and fatigue. Fatigue of materials and structures during the development of modern and advanced technologies is an important topic for probabilistic modeling, and the proposal for this book offers solutions."

As a result of this review report CRC Press agreed to publish the book. Then I was translating and revising the technical issues such as the resolution of the graphs, and putting the latest research results (mainly the extension of the Zhentong Gao method to the MLE to become the generalized Zhentong Gao method (Xu, 2023), and pointing out the reason why it is inappropriate to take the fatigue life data in logarithmic terms (Xu and Gao, 2023) into the English version as well. This English version is actually a revised version of the Chinese version. This process took almost half a year until May 2024, when CRC Press confirmed that the English version of Intelligent Fatigue Statistics would be released worldwide in July 2024 in London.

3. Theoretical Foundations of "Intelligent Fatigue Statistics"

In the introduction, it is pointed out that Mr. Gao's 3 unique insights into the Weibull distribution are in fact the theoretical basis of Intelligent Fatigue Statistics.

1. In the field of fatigue life and reliability the position parameter can be viewed as a safe life with 100% reliability. It cannot be greater than the smallest value in the data, and it will be understood in the next section that the so-called Zhentong Gao method is also built on precisely this property;
2. Weibull distribution is a full-state distribution, in this sense it is more general than the Gaussian distribution, or in a sense the Gaussian distribution is only a first-order approximation of the Weibull. Below is a PDF schema of the Weibull distribution;

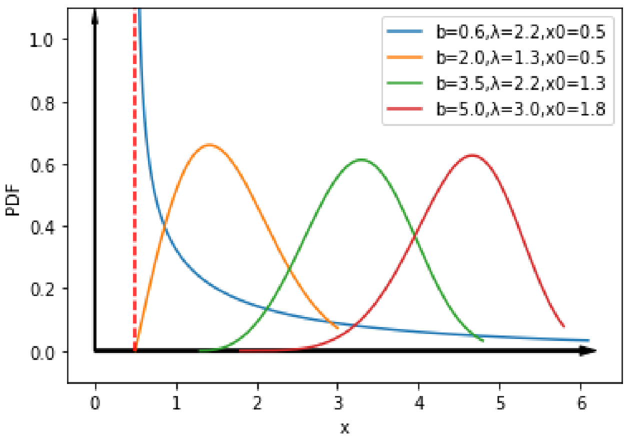

Fig1. *PDF schema of Weibull distribution*

What is clear is that when 0<b<1 it resembles a 1/x function, while 1<b<3 is a left-partial distribution, 3<b<4 approximates a Gaussian distribution, and b>4 is a right-partial distribution. This will be discussed further below. In addition, when b=1, it is an exponential distribution, and when b=2, it is a so-called Rayleigh distribution. This is the main reason why Mr. Gao called the Weibull distribution the "full-state distribution". This also indicates that the Weibull distribution is more general than the Gaussian distribution and will be more widely used.

3. As soon as the asymmetric data is logarithmically more symmetric, it can be fitted with a Gaussian distribution. People get so used to this that they do not notice that this distorts the physical meaning of the data and there is no such thing as a safe lifetime with 100% reliability(Xu and Gao, 2023).

Example1. Using the "Data on gear tooth fracture" in (Gu, 1997), the following results can be obtained by Python code,

fatigue life (original data)=N= [39.6, 40.0, 40.9, 43.6, 43.6, 43.8, 43.9, 44.9, 44.9, 47.4, 47.4, 49.4, 49.6, 52.2, 53.5, 53.7, 54.2, 54.4, 55.0, 56.0, 57.5, 58.4, 61.8, 62.0, 62.5, 62.8, 62.8, 65.7, 67.4, 68.9, 71.8, 72.1, 72.1, 72.1, 72.1, 73.7, 75.8, 75.8, 75.8, 76.5, 82.5, 86.0, 87.1, 89.4, 90.4, 92.0, 95.1, 97.7, 99.7, 101.4, 101.7, 101.7, 103.1, 104.0, 106.1, 106.1, 107.9, 108.8, 109.7, 115.9, 119.9, 122.4, 124.0, 124.0, 125.5, 132.0, 139.5, 155.6, 176.5, 198.5, 232.0] (10^2cycle)

By using the Zhentong Gao method, it can be concluded that

r= 0.99610, b= 1.145, λ= 49.52, N_0= 38.140

Look at the two graphs below. It is clear that the original data is asymmetric and left-partial, but after taking the logarithm it is more symmetric and can be fitted with a Gaussian distribution. Even with a Gaussian distribution it looks like a good fit, but there is no longer a safe life with 100% reliability.

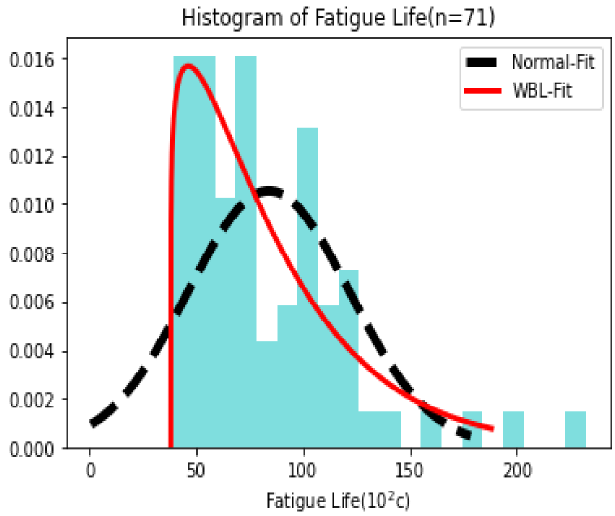

Fig2. *Histograms of raw data before taking logarithms*

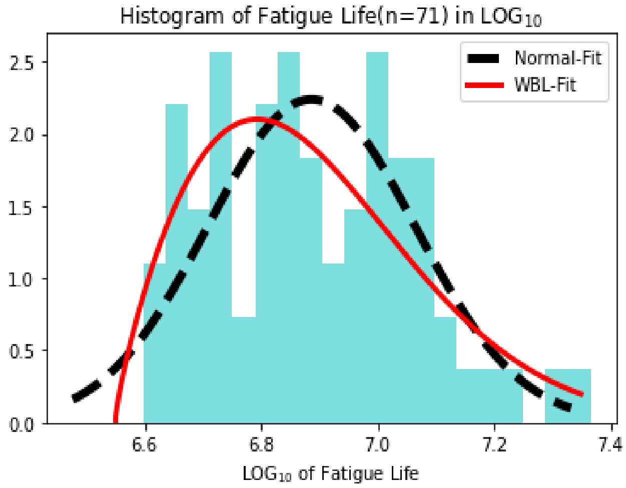

Fig3. *Histogram of raw data after taking logarithms*

These three points form the theoretical basis of “Intelligent Fatigue Statistics”, i.e., Intelligent Fatigue Statistics could not exist without these three unique views of Mr. Gao.

4. An Intelligent Approach to “Intelligent Fatigue Statistics”- - Zhentong Gao Method

Objectively speaking, just with Mr. Gao's three unique insights, there are still some obstacles to the publication of "Intelligent Fatigue Statistics", because of the lack of a convenient method to estimate the three parameters of the Weibull distribution, and if there is no such method, it is quite difficult to use the three-parameters Weibull distribution. This is one of the reasons why Mr. Gao's three unique insights have not been published for decades. On the one hand, I understand these unique insights of Mr. Gao, on the other hand, I also happen to like computers, especially Python is very good to use, so I tried to solve this problem. As a result, I really found a "simple and brutal method" to solve this problem by utilizing the nature of 100% reliable security life and the characteristics of Python, and I named this method as "Zhentong Gao method" (Xu, 2021). From (Xu, 2021), starting from a two-parameter Weibull distribution, note that the definition of reliability is given by

$$P=1-F(x_p/\lambda)=\exp[-(x_p/\lambda)^b] \quad (3)$$

Here P is the reliability, and so (3) after taking the second natural logarithm yields,

$$\ln(\ln(1/p_i))=b\ln(x_i)-b\ln(\lambda) \quad (4)$$

where, $p_i=1-i/(n+1)$。

Set again,

$$Y_i=\ln(\ln(1/p_i)),\ X_i=\ln(x_i) \quad (5)$$

So, $Y_i=bX_i+d$ (6)

where, $d=-b\ln(\lambda)$ and $\lambda=\exp(-d/b)$ (7)

Note that the position parameter x_0 is not negligible, and the parameters b and λ are precisely related to it, according to Mr. Gao's understanding of x_0 it must be in the interval $[0, x_{min})$, where x_{min} is the smallest value in the data set. Therefore, we can calculate the correlation coefficient r corresponding to x_0 by "brute force method" point by point, and the accuracy depends on the actual situation, so that we can determine the x_0 that makes the maximum of r, so, as to determine the corresponding other two parameters b and λ. This is the Zhentong Gao method: "1. Input the original data, if the original data is not sorted, the data is not sorted, the data is not sorted. If the raw data is not sorted, then sort it first. 2. Use Python's scipy to directly traverse the interval $[0, x_{min})$ of possible values of x_0 with a given precision to find the x_0 that maximizes the correlation coefficient, i.e., x_{0max}. 3. Notice that the correlation coefficients in scipy are in fact calculated by first solving for the corresponding coefficients of the equation of a line in the least squares method, b and λ. d andλ=exp(-d/b). Thus, once x_{0max} has been determined, the corresponding parameters b and λ of the Weibull distribution are obtained simultaneously". The advantage is that there is no need for complex mathematical derivations or to solve a system of 3 transcendental equations.The Python code is also very simple and almost obvious immediately(Gao and Xu, 2022).

Example2. Using the data from Example1, which is also the data from (Gu, 1997), we now estimate the three parameters of the fitted Weibull distribution using the Zhentong Gao method,and compare the results with those obtained in (Gu, 1991).Using Python code, it is obtained that

The results obtained by the Zhentong Gao method are as follows:

r= 0.996243, b= 1.143, λ=4949.75, N_0= 3815.00

Correlation coefficient between Gaussian distribution and ideal reliability= 0.98313

Correlation coefficient between Weibull distribution and ideal reliability= 0.99692

Correlation coefficient between Weibull distribution and ideal reliability (Gu (1997))= 0.99637

And the result for (Gu(1997))：b=1.129; λ=4642, N_0=3941

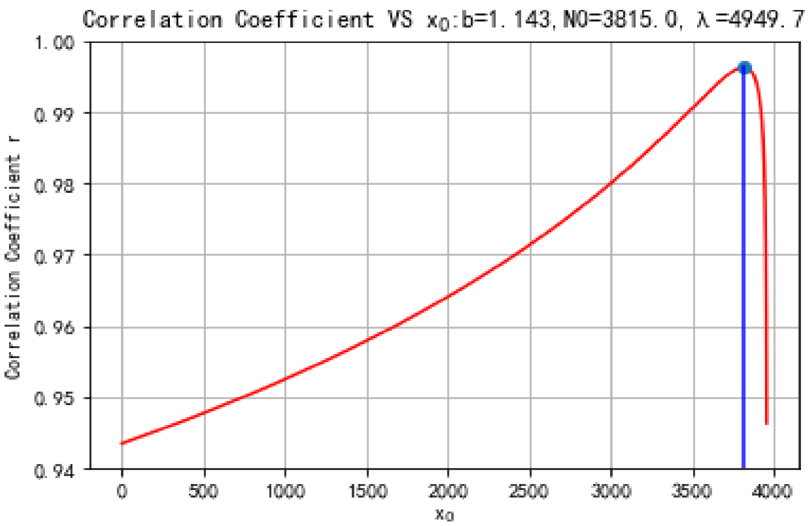

Fig4. *Safety Life vs Correlation Coefficient*

It is not difficult to see that the three parameters of the Weibull distribution obtained by the Zhentong Gao method are significantly better than the results obtained in (Gu, 1997). As for fitting with Gaussian distribution is obviously not as good as fitting with Weibull distribution.

5. Further Development of the Zhentong Gao Method

After the publication of the Chinese version of Intelligent Fatigue Statistics, it was soon found that it could be generalized to the MLE aspect and become the generalized Zhentong Gao method(Xu, 2023).

Example3. Still using the data from Example 1, it can be obtained through Python

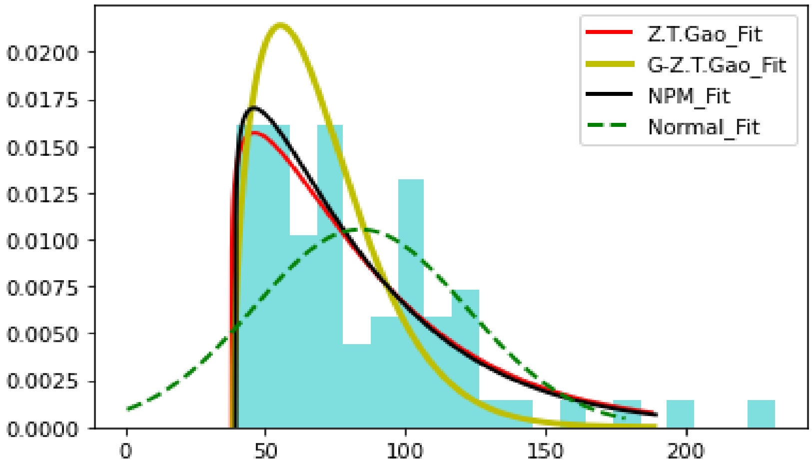

Fig5. *Fitting of distributions obtained by different methods to the data*

The "NPM_Fit" in the figure represents the PDF of the WD drawn with the parameters obtained from the nonlinear programming, which shows that its effect is almost the same as that of the Zhentong Gao method, and it is also better than that of the generalized Zhentong Gao method.

However, both the Zhentong Gao method and the generalized Zhentong Gao method can only estimate the values of three parameters, and there is no way to give their confidence intervals. Later, inspired by the MH (Metropolis-Hastings) algorithm (Metropolis, et al, 1953), it was found that the MH algorithm can be used to upgrade the Zhentong Gao method and generalized Zhentong Gao methodnot only to improve the computational efficiency but also to give the confidence intervals for the three parameters of the Weibull distribution, and more importantly, it can be used in other fields that require optimization computation (e.g., finding the extrema and other mathematical problems related to solving various equations, etc.) (Xu,2024). This discovery is very encouraging, and it is unexpected that the "stochastic method" can "outperform" the "brute force method". The difference between the so-called random method and the brute force method is only in the second step of the Zhentong Gao method, which is no longer to take x_0 point by point, but to take x_0 randomly in the interval $[0, x_{min})$ to calculate the corresponding correlation coefficients. This makes it faster and easier for the computer to find very large correlation coefficients. Hence, I call it the upgraded Zhentong Gao method.

Example4. Still utilizing the data from Example 1, modifying the Python code yields the following results,

γ= 0.95, Overall Sampling Number= 100

b's confidence interval is (1.079, 1.245)av=1.162

λ's confidence interval is (48.663, 50.904)av=49.783

N0's confidence interval is (6.981, 38.871) av=37.926

Zhentong Gao_MC:b=1.162, λ=49.78,N0=37.93 , r=0.99703, LL=-340.835

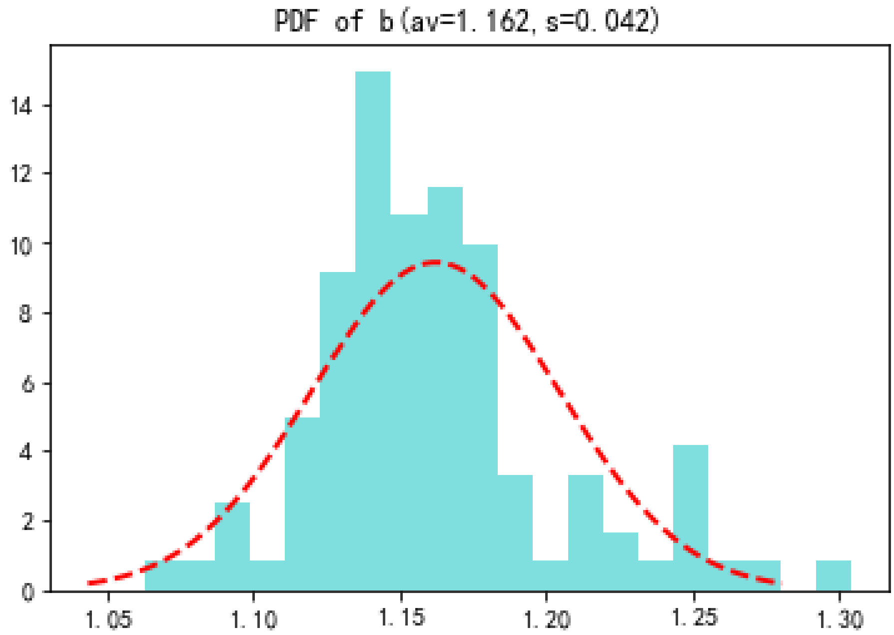

Fig6. *Histogram of the Shape parameter b*

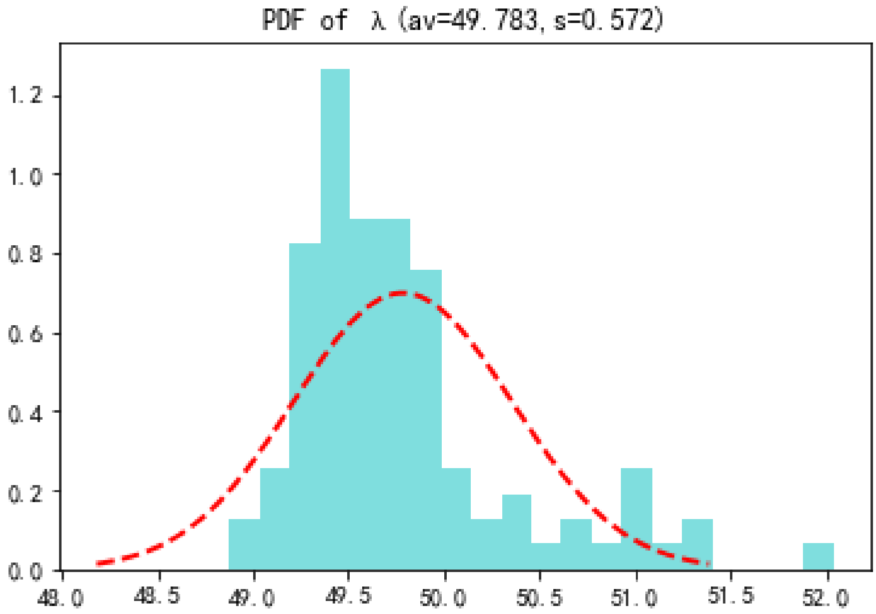

Fig7. *Histogram of the Scalar parameter λ*

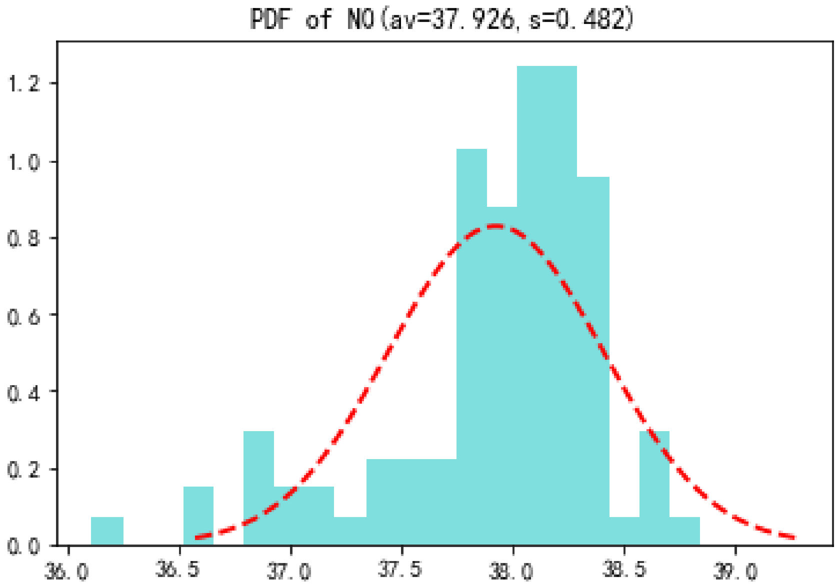

Fig8. *Histogram of the Local parameter N0*

6. Conclusion

Through the above discussion, we can get the following preliminary conclusions:

1. Mr. Gao's three unique insights on Weibull distribution is the theoretical basis of "Intelligent Fatigue Statistics," which is also the key to the birth of the Chinese and English versions; on this basis, combined with the characteristics of Python, it is possible to put forward the Zhentong Gao method, which makes these insights easy to express, and "Intelligent Fatigue Statistics "can be released.

2. The significance of "Intelligent Fatigue Statistics" is that it provides a model for the successful integration of theory and computers (AI).

3. The significance of "Intelligent Fatigue Statistics" also lies in the fact that to realize Mr. Gao's insights, the Zhentong Gao method and the upgraded Zhentong Gao method not only solved the problem of estimating the three parameters of the Weibull distribution and their confidence intervals, but also solved the general mathematical problems such as solving equations and finding the extreme value. That is, in turn, it promotes the development of algorithms.

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References

[1] Gao, Z.T. (1986),Fatigue applied statistics, National Defense Industry Press, Beijing

[2] Gao, Z.T.&Xiong, J.J. (2000),Fatigue Reliability, Beihang University Press, Beijing

[3] Gao Z.T. and Xu J.J. (2022)Intelligent Fatigue Statistics, Beihang University Press, Beijing

[4] Gao Z.T. & Xu J.J. (2025), Intelligent Fatigue Statistics, CRC Press, London

[5] Efron B., Hastie T. (2019), Computer Age Statistical Inference, China Machine Press, Beijing

[6] Fu H.M. & Gao Z.T. (1990), An Optimization Method of Correlation Coefficient for Determining Three-parameters Weibull Distribution,Acta Aeronautica et Astronautica Sinica, 11(7), A323-327

[7] Gu Y.X. (1997), Estimation Methods for three-Parameter Weibull Distribution, Journal of Shenyang Institute of Technology, 16(4)

[8] Hu E.P., Lou X.B. & Liu G.Q. (2000),Estimation Methods for three-Parameter Weibull Distribution,

Journalof Shenyang Institute of Technology, 19(3)

[9] Metropolis, et al. (1953), Equation of State calculation by Fast Computing Machines, J. Chemical Phys, 1953,21: 1087-1091

[10] Xu J.J. (2021), Gao Zhentong Method in the Intelligentization of Statistics inFatigue, Journal of Beijing University of Aeronautics and Astronautics,47(10):2021-2033

[11] Xu J.J. (2022), Digital Experiment for Estimating Three Parameters and Their Confidence Intervals of Weibull Distribution, International Journal of Science, Technology and Society, Vol. 10, No. 2, 2022, pp. 72-81, Doi: 0.11648 /j.ijsts. 20221002.16

[12] Xu J.J.& Gao Z.T.(2022), FurtherResearch on Fatigue Statistics Intelligence,Acta Aeronautica et Astronautical Sinica, 43(8). doi: 10.7527/S1000.6893.2021.25138

[13] Xu J.J.(2023), Generalized Zhentong Gao Method for Estimating Three-Parameters of Weibull Distribution, SCIREA Journal of Computer, 8(2), Doi: 10.54647/computer520345

[14] Xu J.J.& Gao Z.T.(2023) From Gaussian Distribution to Weibull Distribution, Global Journal of Researches in Engineering: I Numerical Methods, Volume 23, Issue 1 Version 1.0, Doi:10.34257/GJREIVOL23IS1PG1

[15] Xu J.J.(2024), Using Monte Carlo method to Upgrade Zhentong Gao method, World Journal of Advanced Research and Reviews, 22(02), 633–640, Doi:10.30574/wjarr.2024.22.2.1391

[16] Yan X.D.， Ma X.， Zheng R.Y.&Wu L(2005), Comparison of the Parameters Estimation Methods for three-Parameter Weibull Distribution, Journal of Ningbo University,18(3)

[17] Zhang P. et al. (2010), Joint Confidence Interval of Parameters of Two –Parameter Weibull Distribution, Mathematical Theory and Applications,3(3)

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