

Determination of Static Bottom Hole Pressure of Dry Gas Wells by a Modified Sukkar and Cornell's Method

Nwankwo, P. C.¹, Nwankwo, K. O.² and Olaleye V.³

¹University of Ibadan, Ibadan, ²Chevron Nig. Ltd, ³University of Ibadan, Ibadan

***Corresponding Author:** Nwankwo, P. C., University of Ibadan, Ibadan.

Abstract: Static Bottom-Hole Pressure (SBHP) plays a vital role in efficient recovery and lifting operations. Knowledge of SBHP is relevant in well testing and is necessary for predicting the productivity or absolute open flow potential in gas wells. However, obtaining precise measurements can be difficult.

An accurate measurement of determining SBHPs can be obtained by use of pressure gauges, however, their application are expensive and sometimes impracticable to apply. Sufficiently accurate estimates can be obtained inexpensively from well head data by evaluating the pressure difference equal to the height of the gas column for static wells, and for flowing wells, friction effects in addition to the weight of the column of gas is required.

A fast, easy, inexpensive and non-iterative estimation method of static bottom-hole pressures in gas wells is that developed by Sukkar and Cornell. This method requires taking readings of integral values from tables to be used to determine pseudo reduced bottom-hole pressures, which is then converted to static bottom-hole pressure. Exiting a program to obtain values from tables disrupts computational flow when executing computer algorithms.

This work aims to eliminate the use of tables by the development of correlations for easy integration in computer programs. The input data were taken by reading from the Sukkar and Cornell's published table of integral values for reduced temperature and reduced pressure ranges of $1.5 < T_{Pr} < 1$ and $1 < T_{Pr} < 10$, respectively.

Two correlations were developed by multiple linear regression for low pressure and high pressure wells, with RMSE of 0.05 and 0.064, respectively. The coefficients of determination of 0.974 for low pressure and 0.973 for high pressure gas wells, showed that the correlations can be used to determine the Sukkar and Cornell integral values with sufficient accuracy. The maximum average absolute error associated with the correlations was 4.77%

The scope of this work was limited to the range of data covered by the Sukkar and Cornell Integral tables for low pressure wells at static conditions ($B = 0$). Application is limited to dry gas reservoirs.

1. INTRODUCTION

Pressure within underground rock formations that hold fluids like oil, gas, or water is referred to as reservoir pressure. Reservoir pressure, measured in pounds per square inch (psi), is a vital factor in assessing and managing hydrocarbon reservoirs. Changes in reservoir pressure can greatly affect how fluids flow through the formation and how easily they can be extracted. Some factors which affect reservoir pressure are depth and breadth of the formation, permeability of rock matrix, pressure at well-head and rock fluid expansion. Conventionally, down-hole reservoir pressure is measured using a Pressure Gauge or sensor. Pressure at the bottom of a wellbore when the well is not producing any fluid is called Static Bottom Hole Pressure (SBHP).

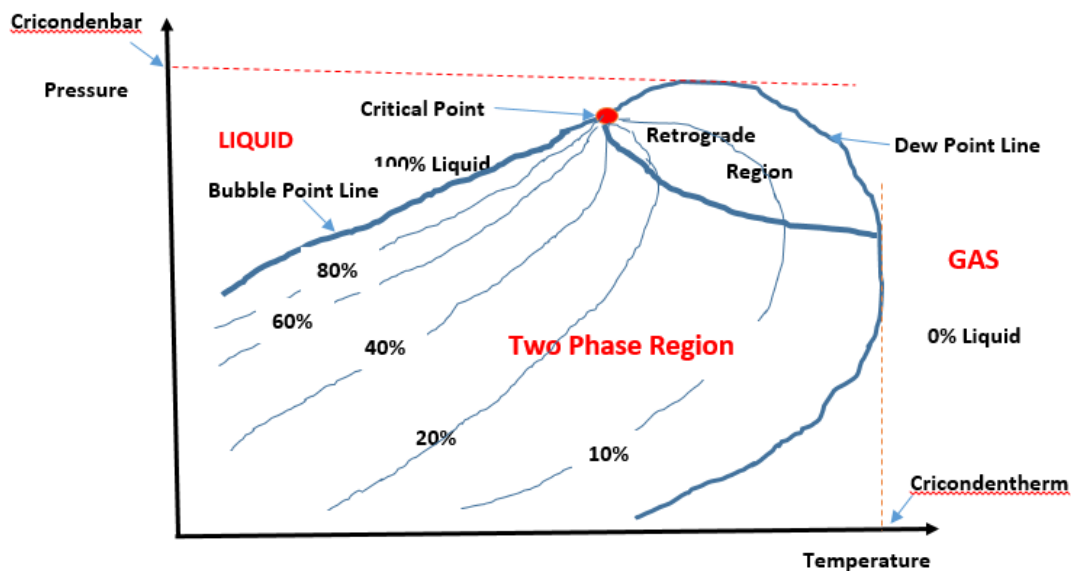


Fig1. Typical P-T diagram for a multi-component system

Gas reservoirs at initial temperature and pressure conditions contain gaseous hydrocarbon wholly, with no liquid hydrocarbon. They plot to the right of the critical point on a Pressure-Temperature phase diagram as shown in figure 1. Gas reservoirs are further classified into three, namely: dry gas, wet gas and gas condensate reservoirs. Dry gas reservoirs, which is the subject of this study, contain predominantly methane with minimal amounts of ethane plus fractions called Natural Gas Liquids (NGLs).

Dry gas reservoirs exist at temperatures much greater than the cricondentherm and as a result, when pressure drops due to production at constant reservoir temperature (isothermal conditions), the reservoir fluid remains in the single phase gas zone, that is retrograde condensation does not occur in the reservoir. The fluid being produced to the surface at decreasing temperature and pressure also has the separator condition falling outside the two phase (oil and gas) region at the single gas phase zone, implying that condensation does not occur within the separators, resulting in minimal or no liquid at separators. (See figure 2 below). Dry gas typically has a high gas-to-oil ratio, often exceeding 100,000 standard cubic feet per stock tank barrel (scf/STB).

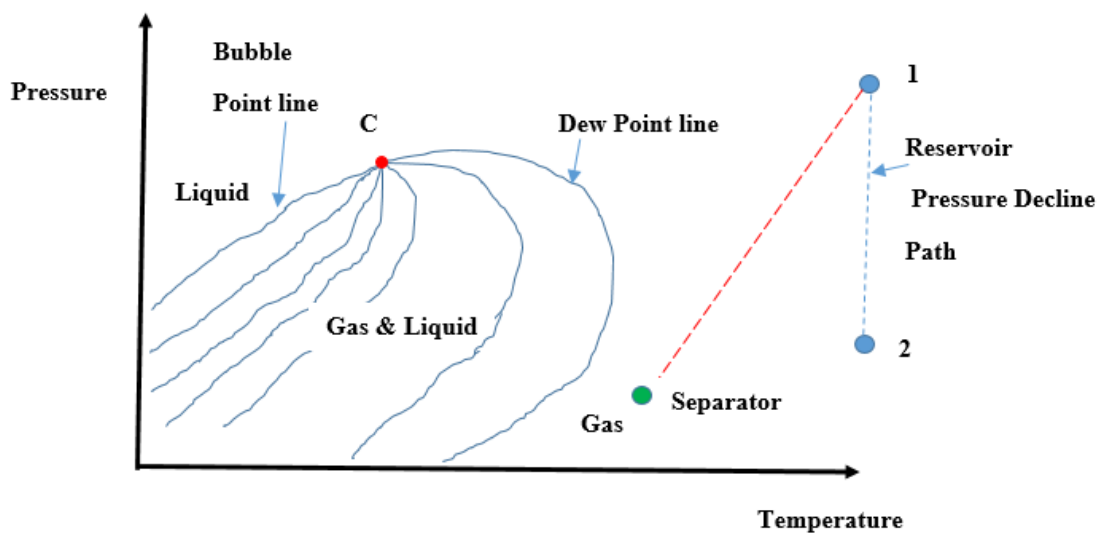
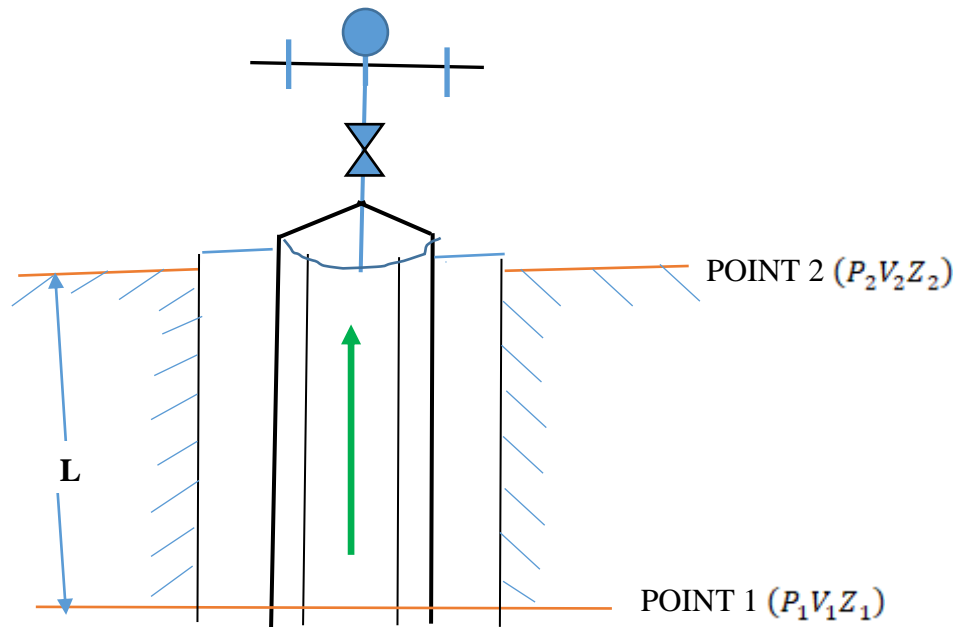


Fig2. P-T Phase diagram of a Dry Gas Reservoir

Monitoring the static bottom hole pressure (SBHP) in a gas well is vital for optimizing production and managing the well effectively. SBHP serves as a key indicator for evaluating the productivity of oil and gas wells, offering valuable insights into the performance of the well and reservoir conditions.

2. THEORETICAL BASIS

The governing equation from which most methods for determining Bottom Hole Pressures (BHP) are derived is the energy conservation equation, or the first law of thermodynamics, which states that, for a fluid, such as gas, moving from point 1 to point 2 as shown in figure 3, the sum of the changes in internal energy, kinetic energy, potential energy and pressure energy equals to zero.



The energy balance between points 1 and 2 for a fluid flowing at steady state may be written as:

$$U_2 + \frac{mV_2^2}{2g_c} + \frac{mg}{g_c}Z_2 + P_2V_2 = U_1 + \frac{mV_1^2}{2g_c} + \frac{mg}{g_c}Z_1 + P_1V_1 + Q - W \quad (2.1)$$

Equation (2.1) can be re-written as:

$$\Delta U + \frac{m\Delta V^2}{2g_c} + \frac{mg}{g_c}\Delta Z + \Delta(PV) = Q - W \quad (2.2)$$

Where, U = internal energy

PV = energy of expansion or compression

$\frac{mV^2}{2g_c}$ = kinetic energy

$\frac{mg}{g_c}Z$ = potential energy

Q = heat energy added to fluid

W = work done on the fluid by the surroundings (for work done by the fluid on the surroundings, W is negative).

Dividing Eqn. (2.2) by m to obtain energy per unit mass balance yields:

$$\Delta U + \frac{\Delta V^2}{2g_c} + \frac{g}{g_c}\Delta Z + \Delta(PV) = Q - W \quad (2.3)$$

Since $\Delta U = \int_{S_1}^{S_2} T dS + \int_{V_1}^{V_2} P(-dV)$ and $\int_{S_1}^{S_2} TUS = Q + E_i$

Where, T = Temperature,

S = entropy and

E_i = irreversible energy losses, and

$$\Delta(PV) = \int_{V_1}^{V_2} P dV + \int_{P_1}^{P_2} V dP$$

Equation (2.3) can thus be expressed in the form:

$$\int_{P_1}^{P_2} V dP + \frac{\Delta V^2}{2g_c} + \frac{g}{g_c} \Delta Z = -W - E_i \quad (2.4)$$

For the flow illustrated in Figure 3, there is no work done by or on the flowing fluid, therefore, W is equal to zero and equation (2.4) reduces to:

$$\int_{P_1}^{P_2} V dP + \frac{\Delta V^2}{2g_c} + \frac{g}{g_c} \Delta Z = -E_i \quad (2.5)$$

For a static fluid columns, equation (2.5) reduces to:

$$\int_{P_1}^{P_2} V dP + \frac{g}{g_c} \Delta Z = 0 \quad (2.6)$$

Or

$$\int_{P_1}^{P_2} \frac{dP}{\rho} + \frac{g}{g_c} \Delta Z = 0 \quad (2.7)$$

Since $\frac{g}{g_c}$ is assumed to be unity, Equation (2.7) is then reduced to:

$$\int_{P_1}^{P_2} \frac{dP}{\rho} + \Delta Z = 0 \quad (2.8)$$

For a vertical pipe flow as illustrated by figure 3, $\Delta Z = L$, where L is the length of the pipe string, so that equation (2.6) can be written as:

$$\int_{P_1}^{P_2} V dP = L \quad (2.9)$$

$$\text{From the real gas equation of state, } PV = znRT, \quad (2.10)$$

where, n is number of moles = m/M ,

M = molecular weight,

m = mass of gas,

R = gas constant, and

z = gas deviation factor (also called gas compressibility factor)

$$\text{the volume per unit mass is given by } V = \frac{zRT}{PM} \quad (2.11)$$

Substituting Equation (2.11) into (2.9) gives:

$$\int_{P_1}^{P_2} \frac{zRT}{M} \frac{dP}{\rho} = L \quad (2.12)$$

$$\text{Given that gas specific gravity, (air =1) is given by, } \gamma_g = \frac{M}{M_{air}} \quad (2.13)$$

Where, M is the molecular mass of gas and M_{air} is molecular weight of air which is approximately, **28.976 lb m/lb mol**, For a particular gas,

$$\frac{R}{M} = \frac{53.241}{\gamma_g} \quad (2.14)$$

where γ_g is a constant for a given gas. Therefore, Equation (2.12) can be expressed in terms of Equation (2.14) as:

$$\frac{53.241}{\gamma_g} \int_{P_1}^{P_2} zT \frac{dP}{\rho} = L \quad (2.15)$$

Equation (2.15) is the starting point from which various methods of solution for calculation of Bottom Hole Pressures derive. The calculation methods differ by the assumptions which are made towards providing a solution to this equation. Assumptions are made in regard to the compressibility factor, z , and temperature, T .

For any calculation method, four surface properties must be known, namely: well-effluent composition or gas gravity, well-depth, wellhead pressure, and temperature.

Several methods of solution can be derivable from Equation (2.15) by making other simplifying assumptions and many have been presented in literature.^{1,4,5,6,7,8}

The work of Sukkar and Cornell (1955) established that the general vertical flow equation can be expressed as:

$$\int_{P_{pr_1}}^{P_{pr_2}} \frac{\left(\frac{z}{P_{pr}}\right) d P_{pr}}{1 - \frac{667 f q_g^2 T^2 z^2}{D^5 P_{pc}^2}} = \int_{P_{pr_1}}^{P_{pr_2}} \frac{\left(\frac{z}{P_{pr}}\right) d P_{pr}}{1 + \frac{B z^2}{P_{pr}^2}} = -0.01877 \gamma_g \int_{L_1}^{L_2} \frac{dL}{T} \quad (2.16)$$

or

$$\int_{P_{pr_1}}^{P_{pr_2}} \frac{\left(\frac{z}{P_{pr}}\right) d P_{pr}}{1 + B \left(\frac{z}{P_{pr}}\right)^2} = -0.01877 \gamma_g \int_{L_1}^{L_2} \frac{dL}{T} \quad (2.17)$$

Where,

$$B = \frac{667 f Q^2 T^2}{D^5 P_{pc}^2}$$

P = pressure, psia

q_g = gas flow rate, MMscfd, referred to at standard conditions of 14.65 psia and 60° F

D = inside diameter of pipe, in.

T = Temperature through distance L , °R

$L = H$ = vertical distance of interest, ft

γ_g = gas specific gravity, no unit

f = friction factor, dimensionless

P_{pc} = pseudo critical pressure, psia

P_{pr_2} = pseudo reduced wellhead pressure, no unit

P_{pr_1} = pseudo reduced bottom hole pressure, no unit

z = gas deviation factor of gas (well effluent) at \bar{T} and at the varying pressures within the flow string

A constant average temperature along the length of pipe was then assumed, which permits integration of the Right Hand Side (RHS) of Equation (2.17) to give:

$$\int_{P_{pr_1}}^{P_{pr_2}} \frac{\left(\frac{z}{P_{pr}}\right) d P_{pr}}{1 + B \left(\frac{z}{P_{pr}}\right)^2} = -\frac{0.01877 \gamma_g}{T} \int_{L_1}^{L_2} \frac{dL}{T} \quad (2.18)$$

For the static case, $B = 0$ and Equation (2.18) reduces to:

$$\int_{P_{pr_2}}^{P_{pr_1}} \left(\frac{z}{P_{pr}} \right) dP_{pr} = \frac{0.01877 \gamma_g L}{\tau} \quad (2.19)$$

The limits of integration have been reversed to give the value on the RHS a positive sign.

Sukkar and Cornell (1955) integrated the entire Left Hand Side (LHS) of Eq. (2.19) numerically at several values of pseudo reduced temperature, pseudo reduced pressure and the factor, B and presented their results as tables available in literature. Sukkar and Cornell presented two tables, one for low pressure range of reduced pressures from the range of 1.0 to 5.0 (or pressures within the range of about 600 psia to 3,200 psia). The second table presented by Sukkar and Cornell was prepared for high pressure range of reduced pressures from 3.0 to 12.0, or for pressures above 2,800 psia.

3. MODEL DEVELOPMENT

Model development was based on Linear regression approach where the dependent variable is the value of the Sukkar and Cornell integral at $B = 0$, while the independent variable are the reduced pressures and reduced temperatures. The fundamental structure of a linear regression model can be expressed as:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n + \epsilon \quad (3.1)$$

Where

The dependent variable is denoted by the symbol Y, and

X_1, X_2, \dots, X_n are the independent variables.

The coefficients $\beta_0, \beta_1, \beta_2, \dots, \beta_n$ describe the magnitude and direction of the relationship between the independent variables and the dependent variable.

The symbol ϵ , is the error term. It quantifies the discrepancy between the actual value and the value predicted by the model. This value must be minimized to ensure minimal bias between calculated and estimated values to ensure effectiveness of model.

For the development of the correlation, several linear arrangements of P_{pr} and T_{pr} leading to the estimation of the Sukkar and Cornell integral, I , were suggested and tried for least error estimation and a final arrangement of the form Eq. (3.2) was obtained

$$I = aP_{pr} + b \left(\frac{P_{pr}}{T_{pr}} \right) + c(T_{pr}P_{pr}) + d \quad (3.2)$$

Where a, b, c, d are the coefficients of the correlation for the Sukkar and Cornell integral (I), T_{pr} and P_{pr} are pseudo-reduced temperature and pressure respectively, with d being the correction term. All the new values obtained by division and multiplication ($\left(\frac{P_{pr}}{T_{pr}} \right)$ and $(T_{pr}P_{pr})$), respectively, were the transformed data points.

Data Selection: Values of Sukkar and Cornell's integral values from the low pressure table and the high pressure range of pressure table were systematically read and recorded at values of $B = 0$, which represents static or no flow conditions. Two thousand five hundred and forty (542) and two thousand and eighty four (584) combinations of the Sukkar and Cornell integral values were read from the table of integrals available in literature for the low and high pressure ranges of data, respectively. For the lower reduced pressure range, values from 1.0 to 3.0 were selected, whereas, for the higher reduced pressure range, values between 2.0 and 10.0 were selected, with reduced temperatures between 1.5 and 1.7 for both ranges. Table 3.1 is a summary of the input and output variables for this study.

Table 3.1 Summary of the input and output data for this study (at $B=0$)

	Variables	Minimum	Maximum	Range
Input	Pseudo-reduced Pressure (P_{pr})	1.0	10.0	9.0
	Pseudo-reduced Temperature (T_{pr})	1.5	1.7	0.2
Output	Sukkar & Cornell's Integral value, (I)	0.2343	1.4235	1.1292

Data Splitting: Each of these data sets was shared into three sets in the ratio 3:1:1 for model training, model validation and model testing, respectively

Model Training and Testing: The linear regression model was applied to the training data and error minimization sought by setting d to zero and altering the coefficients, $a, b, \text{ and } c$. This entails the process of approximating the coefficients (parameters) of the model to minimize the discrepancy between the observed and anticipated values. The trained model was tested with the training dataset to check for correctness of the developed model.

Evaluation: The error metrics such as root mean square error, mean absolute percentage error and coefficient of determination were determined to evaluate the performance of the developed models. Using the percentage of data set aside for testing.

Application of this method is verified by the inclusion of a simple problem algorithm for finding the Shut in Bottom Hole Pressure and a computer program in PYTHON language which executes the algorithm are included as Appendices III and IV, respectively.

4. RESULTS AND DISCUSSION OF RESULTS

The optimized linear models for approximating the Sukkar and Cornell integrals for the low and high pressure well conditions, are, respectively:

$$I = 2.97P_{pr} - 2.80 \left(\frac{P_{pr}}{T_{pr}} \right) - 1.06T_{pr}P_{pr} + 1.82 \quad (4.1)$$

and

$$I = 0.10T_{pr}P_{pr} + 0.23 \left(\frac{P_{pr}}{T_{pr}} \right) - 0.47P_{pr} + 1.78 \quad (4.2)$$

Equation (4.1) estimates I for low pressure gas wells with the P_{pr} from 1.0 to 3.0 and T_{pr} from 1.5 to 1.7,

While Equation (4.2) estimates I for pressure high pressure gas wells with the P_{pr} from 2.0 to 10.0 and T_{pr} from 1.5 to 1.7. Samples of results which compare the calculated and estimated integral values (I) for low pressure gas wells and high pressure gas wells are shown as Tables 4.1 and 4.2 respectively. Plots which offer visual assessment of the performance of these various correlations are included as figures 4.1 and 4.2 for the low pressure and high pressure ranges, respectively.

Table 4.1 Example of Actual and Estimated Sukkar and Cornell integral values for low pressure gas wells

Ppr	Tpr	Estimated I	Actual I
1	1.5	1.333	1.4236
1.1	1.5	1.2850	1.338
1.2	1.5	1.236	1.2606
1.3	1.5	1.187	1.1902
1.4	1.5	1.139	1.1255
1.5	1.5	1.09	1.0659
1.6	1.5	1.0413	1.0107
1.7	1.5	0.9927	0.9603
1.8	1.5	0.944	0.9113
1.9	1.5	0.8953	0.8662
2	1.5	0.8467	0.8238

Determination of Static Bottom Hole Pressure of Dry Gas Wells by a Modified Sukkar and Cornell's Method

Fig4.1 The line of actual versus line of predicted values of Sukkar and Cornell integral for low pressure gas wells.

The seemingly large deviation observed at the higher end of the line is due to predicted values for $P_{pr} = 1.0$ and 1.1 at $T_{pr} = 1.5$. Because the $P_{pr} = 1.0$ is a unit value and $P_{pr} = 1.1$ is closer, there is expected higher disparity between the estimated and actual integral since every other reduced pressure values the model estimates the integral for mostly non-unit values or values farther from unit values.

Table 4.2 Example Actual and estimated Sukkar and Cornell integral values for high pressure gas wells

Ppr	Tpr	Estimated I	Actual I
3.1	1.7	1.2694	1.3082
3.2	1.7	1.2529	1.2809
3.3	1.7	1.2365	1.2544
3.4	1.7	1.22	1.2288
3.5	1.7	1.2035	1.2039
3.6	1.7	1.1871	1.1798
3.7	1.7	1.1706	1.1563
3.8	1.7	1.1541	1.1336
3.9	1.7	1.1376	1.1113
4	1.7	1.121	1.0897

Actual Values

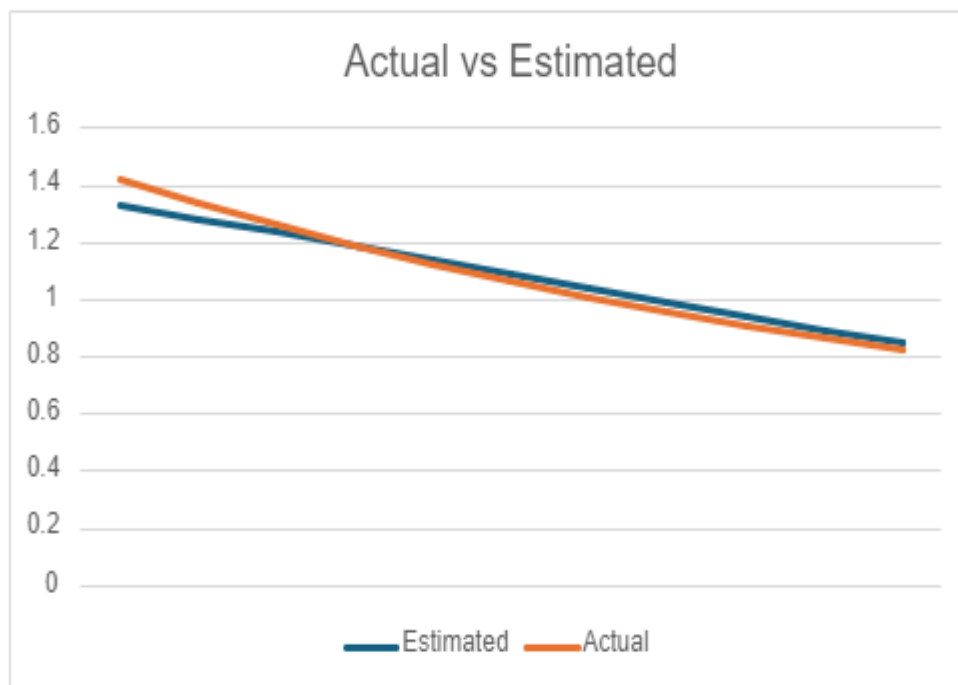


Fig4.2. Comparison of actual versus predicted values of Sukkar and Cornell integral for high pressure gas wells.

From figure 4.2, there is better agreement between the actual and estimated values of the Sukkar and Cornell's integrals for high pressure wells as seen in figure 4.2 above. This indicates that the developed correlation for high pressure wells was able to estimate the Sukkar and Cornell integral with less minimal error when compared to the performance of the correlation for lower pressure integrals.

4.1 Error Analysis

Error metrics (Mean Absolute Percentage Error, Root Mean Square Error and Coefficients of Determination, R^2) were used to evaluate the performance of the developed models. The performance for the low pressure and high pressure well correlations are shown below in Table 4.3.

The statistical formulae used for estimating the errors are included as Appendix I.

Table 4.3. Error metrics for the Sukkar and Cornell integral correlation for low and high pressure gas wells

Error Metric	Low Pressure correlation performance	High Pressure Correlation Performance
Mean Absolute Percentage Error	0.047	0.083
Root Mean Square Error	0.05	0.064
Coefficient of Determination	0.974	0.973

5. CONCLUSIONS AND RECOMMENDATIONS

The application of Sukkar and Cornell's method is straight forward and requires no trial and error. It is accepted as sufficiently accurate for engineering applications and is recommended for gas wells with bottom hole pressures less than 10,000 psi (the approximate pressure limit of Sukkar and Cornell's tables)

The developed correlations can be used to estimate the approximate value of the Sukkar-Cornell Integral with minimal errors, explicitly, and without trial-and-error methods. This is necessary to eliminate loss of time associated with table look up and interpolation between measured values.

The correlations developed for low pressure and high pressure wells both gave coefficient of determination (R^2) greater than 95% and low errors (RMSE and AAPE) which prove that these correlations can be used conveniently to estimate the integral values as represented by the Sukkar and Cornell's table for low and high reduced pressure conditions.

This study makes it easy to use computer programming approach that would be executable easily without losing time to obtain values from external look up tables, which lowers computational efficiency. The method is limited to gas wells with no non-hydrocarbon impurities, or at best, less than 5% mole percent of combined non-hydrocarbon species in mixture.

It is recommended that further work be carried out using more data points and further trials of linear and multiple regression approaches. This could provide a correlation that could further reduce the bias between predicted and actual values. Also more data sets could be obtained and used for model training beyond the limits of pseudo reduced pressures used for this study, ($1 < P_{pr} < 10$), to broaden the range of applicability of the correlations.

REFERENCES

- [1] Fowler, F. C.: "Friction Factors for Pipe Flow", Trans., ASME (1944) 66, 671
- [2] Ikoku .C. U. : "Natural Gas Production Engineering", Reprint edition, (p. 317 – 330) Krieger Publishing Company.1992
- [3] Nwankwo, P. C.: "A Novel Correlation for Better Estimation of Pseudo-Critical Pressures and Pseudo-Critical Temperatures in Natural Gas Systems", International Journal of Science and English. 2020
- [4] Moody, L. F. : "Friction Factors for Pipe Flow", Trans. ASME (1944), 66,671
- [5] Poettmann, F. H.: "The calculation of Pressure Drop in the Flow of Natural Gas Through Pipe", Trans., AIME (1951) 192, 317-24
- [6] Rzasa, M. J. and Katz, D. L.: "Calculation of Static Pressur Gradients in Gas Wells", Trans., AIME (1945) 160, 100 - 06
- [7] Sukkar, Y. K. and Cornell, D.: "Direct Calculation of Bottom Hole Pressures in Natural Gas Wells, Trans., AIME (1955) 204, 43- 48.

- [8] Udoma, C. E., Okon, A. N., Udoeyop, S. U. and Okologume, W. C.: "New Insights of Estimating Static Bottom-Hole Pressures of Gas Wells in Nigerian Petroleum Provinces." *International Journal of Petroleum Technology*,(2023), 135-150

Citation: *Nwankwo, P. C., et al. (2024). "Determination of Static Bottom Hole Pressure of Dry Gas Wells by a Modified Sukkar and Cornell's Method", International Journal of Petroleum and Petrochemical Engineering (IJPPE), 9(1), pp.38-47, DOI: <https://doi.org/10.20431/2454-7980.0901005>.*

Copyright: © 2024 Authors. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited