

# Minimum Added Energy to Ground State of Hydrogen Atom

Janez Špringer\*

Cankarjeva cesta 2, 9250 Gornja Radgona, Slovenia, EU

\*Corresponding Author: Janez Špringer, Cankarjeva cesta 2, 9250 Gornja Radgona, Slovenia, EU

Abstract: The minimum added energy to the ground state of hydrogen atom was calculated.

Keywords: ground state of hydrogen atom, inverse fine structure constant, minimum added energy

## **1. INTRODUCTION**

[1] The total energy of the electron in the ground state of hydrogen atom equals the negative value of Rydberg constant  $-Ry = -13.605\ 693\ 122\ 990\ eV$  and the orbit length expressed in Compton wavelengths of the electron  $s_{orbit}$  equals the inverse fine structure constant  $\alpha^{-1} = s(137)$ . To the ground state added energy  $E_{added}$  is proposed to extend the elliptic orbit length n = 137 for only the whole number of Compton wave lengths of the electron  $n \in \mathbb{N}$  which appears as the average elliptic-hyperbolic length s(137 + n). The absolute value of total energy  $E_{total} = -Ry + E_{added}$  and the extended orbit length  $s_{orbit} = s(137 + n)$  are in inverse proportion. Therefore, we can write the next formula:

$$\alpha^{-1}(-Ry) = s(137)(-Ry) = s(137+n)(-Ry + E_{added}) = constant.$$
 (1)

Here on the continuous double surface holds

$$s(137+n) = (137+n) \left(2 - \frac{1}{\sqrt{1 + \left(\frac{\pi}{137+n}\right)^2}}\right), \ n = 0,1,2,3...$$
(2)

And on the polygonal double surface with N number of polygon sides the number  $\pi$  is replaced by pseudo  $\pi^*$ :

$$\pi^* = N \sin \frac{\pi}{N}.$$
(3)

## 2. MINIMUM ADDED ENERGY TO THE GROUND STATE OF HYDROGEN ATOM

The minimum added energy to the ground state of hydrogen atom is determined by extending the orbit length s(137) by one Compton wavelength of the electron n = 1. Applying the equation (1) we can write:

$$E_{added} = Ry \left( 1 - \frac{s(137)}{s(137+n)} \right).$$
(4*a*)

And on the continuous double surface for n = 1 the next minimum added energy value is given:  $E_{added n=1}^{continuous} = 0.098\ 540\ 750\ 514\ eV.$ (4b)

# 3. TOTAL ENERGY AT MINIMUM ADDED ENERGY TO THE GROUND STATE OF HYDROGEN ATOM

The minimum added energy to the ground state of hydrogen atom yields too small total energy  $E_{total n=1}^{continuous} = -Ry + E_{added n=1}^{continuous} = -13.507\ 152\ 372\ 476\ eV$  which is not stable and cannot emit spectral lines. Since to do so the square root of total energies quotient should equal at least the whole number 2, but obviously it does not as follows:

$$\sqrt{\frac{-Ry}{E_{total\,n=1}^{continuous}}} = \sqrt{\frac{-Ry}{-Ry + E_{added\,n=1}^{continuous}}} = \sqrt{\frac{-13.605\ 693\ 122\ 990\ eV}{-13,507\ 152\ 372\ 476\ eV}} = 1,0036 \neq 2.$$
(5)

# 4. MINIMUM ADDED ENERGY TO THE GROUND STATE OF HYDROGEN ATOM ON THE 129-POLYGONAL SURFACE

On the polygonal double surface with 129 polygon sides the number  $\pi$  is replaced by the pseudo  $\pi_{129}^*$ :

$$\pi_{129}^* = 129 \sin \frac{\pi}{129} = 3.141\ 282\ 121\ 798\ 650\ \dots \tag{6}$$

Yielding with the help of equation (4a) the next minimum added energy to the Ground State of hydrogen atom on 129- polygonal surface:

$$E_{added\ n=1}^{polygonal\ 129} = \mathbf{0}, \mathbf{098\ 540\ 760\ 630\ eV}.$$
(7)

The minimum added energy to the ground state of hydrogen atom on the continuous and polygonal surface differs only on the seventh decimal place.

## 5. ESTIMATION OF MINIMUM ADDED ENERGY TO THE GROUND STATE OF HYDROGEN ATOM

Since the elliptic orbit length n and the average elliptic-hyperbolic orbit length s(n) are very close to each other the minimum added energy to the ground state of hydrogen atom can be estimated as follows:

$$E_{added}^{estimated} \approx Ry\left(1 - \frac{137}{138}\right) = 0,099 \ eV.$$
(8)

#### 6. CONCLUSION

An optimal adsorption energy close to zero (like  $\approx 0.099 \, eV$  in our case) indicates that the proper catalyst can efficiently adsorb hydrogen atoms while also allowing them to desorb easily to form hydrogen gas.[2] The orbit extension in the ground state of hydrogen atom could provide a theoretical basis for the event.

#### REFERENCES

[1] Janez Špringer. " By Polygonal Path to Exact Inverse Fine Structure Constant" International Journal of Advanced Research in Physical Science (IJARPS), vol 11, no. 12, pp. 1-2, 2024.

[2] Chen, Z.W., Li, J., Ou, P. *et al.* Unusual Sabatier principle on high entropy alloy catalysts for hydrogen evolution reactions. *Nat Commun* **15**, 359 (2024). https://doi.org/10.1038/s41467-023-44261-4

**Citation:** Janez Špringer. " Minimum Added Energy to Ground State of Hydrogen Atom" International Journal of Advanced Research in Physical Science (IJARPS), vol 12, no. 01, pp. 21-22, 2025.

**Copyright:** © 2025 Authors. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.