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Adaptive Moving Mesh Finite Volume Method Experimentation to Solving Riemann's Type Hyperbolic Problems

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Abstract: *The purpose of this paper is to report once more on the benefits of using an adaptive moving meshmethod with finite volume method through the numerical resolution of two test problems. This is Buckley-Levrett's problem and a broadcast transport problem. The peculiarity of these problems isthat they consist of hyperbolic Partial Derivative Equation (PDE) in conservative form and solutions at the initial moment which are discontinuous. To reach the set objective, we presented the selected problems and the adaptive moving mesh method that will be used with finite volumemethod for their numerical resolution. The results obtained have been validated through several evaluations. Firstly, we determined and validated for each problem the numerical solution of the finite volume method with a static grid by evaluating it with respect to that determined by the finite difference method. then, in a second step, this solution of the finite volume method with a static grid served as a reference solution to evaluate the numerical solution of the finite volume method with a moving grid.*

Keywords: *Hyperbolic conservation laws, Riemann problem, classical finite volume method, higher order finite volume method, adaptive moving mesh method under the finite volume method.*

1. Introduction

Nowadays, one of the most used mathematical branches in the engineering field is numerical analysiswhich is a branch of applied mathematics, the aim of which is to develop and study numerical resolution methods from physics, applied sciences or social and economic models. one of the most recent approaches in the numerical resolution of PDEs is the adaptive moving mesh method usedunder the different numerical methods what use a discretization of the problem solving domain. Namely the finite difference method, the finite element method and the most recent of all which is the finite volume method. the adaptive moving mesh method is used in the resolution of Ordinary Differential and Partial Differential Equations ODEs and PDEs) in order to improve the accuracyof the solution in areas where the gradient of this one has high values. The development of this technics began with the work of Harten and Hyman in 1983. With the same idea, Winslow also made a proposition in [15] on the finite element method. This proposition was taken up and improved by Dvinsky in [6] in 1991. Thus, Saleri and Steinberg proposed a variant in 1994. The technical leaders discussed in this document are T. TANG and T. Huazhong (Chinese). They were followed by other research teams such as Zegeling and A. VANDAM (Dutch). In order to better imbibe this new numerical mathematical problem solving technics, we set out to experiment it for the resolution of two Riemann's type hyperbolic problems by the finite volume method. This paper is organized as follows. Section 2 presents the test problems on which the experience will be focused. Section 3 presents the adaptive moving mesh method used under the high-order finite volume method. Section 4 reports on the validation of the numerical results obtained through a series of comparisons

2. DESCRIPTION OF MODEL PROBLEMS

2.1. Buckley-Leverett equation

The equation provides a simplified model of the horizontal scanning of an oil test tube by water whileneglecting capillarity [9]. The model is characteristic of the situations encountered in petroleum operations [5, 9]. There is water on the left and oil on the right. We search to drive the oil to the right thanks to the water which is injected by an injection tube.

The problem considered is the following: let u be the water saturation in a mixture of water and oil,and we consider a 1D cut from the basement. Find the water saturation u in a mixture of water and petroleum such that:

$$\partial_t u + \partial_x (f(u)) = \varepsilon \frac{\partial^2 u}{\partial x^2} \quad \text{where} \quad f(u) = \frac{u^2}{u^2 + A(1-u)^2}$$

where $A > 0$ is the parameter. At the initial time

$$u_0(x) = \begin{cases} 1 & \text{if} \quad x \in [0, l[\\ 0 & \text{if} \quad x \in [l, L]. \end{cases}$$

The initial data corresponds to saturated water on the left and oil on the right. As it there is a source of water injection, the condition at the edges in 0 is maintained at saturation state $u(0,t) = 1$, similarly the condition at the edges L is considered to be an unsaturated state at each instant i.e. $u(L,t) = 0$.

2.2 Transport-diffusion equation

The transport-diffusion problem [?, ?] is defined by: determining in a fluid medium little deep at an assumed known and constant velocity u, the concentration of the pollutant $C(x,t)$ solution of:

$$\partial_t C + \partial_x (CU) = D \frac{\partial^2 C}{\partial x^2} \tag{1}$$

With $U > 0$ a constant velocity, D the diffusion coefficient of the pollutant. At the initial time $C(x,0) = C_0(x) = \begin{cases} 2 & \text{if} \quad x \in [0, l[\\ 0 & \text{if} \quad x \in [l, L]. \end{cases}$

3. DESCRIPTION OF THE USED MOVING MESH FINITE VOLUME METHOD

In recent years, much work has been done on the adaptive moving mesh. In general, the numerical resolution of a problem by a numerical method with a moving mesh is done in two steps: the setting up of the moving mesh and the resolution of the PDE of the problem (also called physical PDE) on this mesh. For this work, we use the technique proposed by T. TANG and Huazhong TANG in [19] where the numerical method chosen for the resolution of the PDE is finite volumes.

3.1. Description of the Strategy Used to Obtain the Moving Mesh

The space interval is subdivided into elementary intervals at the initial time according to the finite volume method to obtain the nodes of the moving mesh where the solution values will be computed. The nodes resulting from this subdivision form the components of a vector X. The position of thecomponents of the vector X are updated according to the solution gradient values. The relationship between the mesh movement and the solution is established by a function called control or monitorfunction ω recalled in [17, 2]. The vector X is defined as:

$$X \; : \; [0,1] \; \rightarrow \; [a,b] \; , \qquad \xi \rightarrow x(\xi)$$

Where X is the solution of the moving mesh PDE defined by:

$$\begin{cases} \omega(X_\xi)_\xi & = 0 \\ X(0) & = a \\ X(1) & = b \end{cases} \tag{2}$$

There are several variants of the moving mesh PDE and there are also several possible choices for the monitor function. The one we used in this work according to, [?, ?] is defined by:

$$\omega(U) = \sqrt{1 + \alpha \|U_x\|_2^2} \quad ,$$

where U_x must be approximated by a centered finite difference scheme, and α is a no negative real parameter. In this case it is written:

$$\omega(U) = \sqrt{1 + \alpha \left(\frac{U_{i+1}^n - U_{i-1}^n}{\Delta_i} \right)^2} \quad ,$$

$x_i^n = \frac{1}{2}(x_{i-\frac{1}{2}} + x_{i+\frac{1}{2}})$ and $\Delta_i^n = \frac{1}{2}(x_{i+\frac{3}{2}} - x_{i+\frac{1}{2}}) + (x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}) + \frac{1}{2}(x_{i-\frac{1}{2}} + x_{i-\frac{3}{2}})$.

The moving mesh PDE discretization according Gauss-seidel gives the numerical scheme used by Mackenzie in [?, ?]. According to this scheme, over time, the position of the components of X are determined by:

$$X_{i+\frac{1}{2}}^{n+1} = \varphi_{i+1}^n X_{i+\frac{3}{2}}^n + (1 - \varphi_{i+1}^n - \varphi_i^n) X_{i+\frac{1}{2}}^n + \varphi_i^n X_{i-\frac{1}{2}}^n$$

where $\varphi_i^n = \frac{\Delta\tau}{(\Delta\xi)^2}\omega(U_i^n)$, with $\Delta\tau$ such a value that $\max(\varphi_i^n) \leq \frac{1}{2}$.
To get better control of the grid distribution near those regions where the solution has a large gradient, according to the solution stiffness, it is necessary to perform several iterations according to the system formed by the previous equation and the next:

$$U_i^{n+1} = \frac{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}}^{n+1} - x_{i-\frac{1}{2}}^{n+1}} - \frac{1}{x_{i+1}^{n+1} - x_{i-\frac{1}{2}}^{n+1}} \left((cu)_{i+\frac{1}{2}}^n - (cu)_{i-\frac{1}{2}}^n \right) \tag{3}$$

with

$$(cu)_{i+\frac{1}{2}}^n = \frac{C_{i+\frac{1}{2}}}{2} \left(U_{i+\frac{1}{2}}^{n+} - U_{i-\frac{1}{2}}^{n-} \right) - \frac{|C_{i+\frac{1}{2}}|}{2} \left(U_{i+\frac{1}{2}}^{n+} - U_{i-\frac{1}{2}}^{n-} \right) \tag{4}$$

with $C_{i+\frac{1}{2}} = x_{i+\frac{1}{2}}^n - x_{i+\frac{1}{2}}^{n+1}$

$$U_{i+\frac{1}{2}}^{n-} = U_i^n + \tilde{\psi}_i^n (x_{i+\frac{1}{2}} - x_i)$$

$$U_{i+\frac{1}{2}}^{n+} = U_i^{n+1} + \tilde{\psi}_{i+1}^n (x_{i+1} - x_{i+\frac{1}{2}})$$

where $\tilde{\psi}$ is a flux limiter that can be defined in several ways. In fact, the scheme ?? allows to ensure the update of the values of the physical PDE solution after each change in the position of the nodes over time. The passage of a node position to another takes into account the conservation of the total mass at each instant t^n. The each instant t^n , the new node positions are used to define the elementary volumes $V_i^n = \left] x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}} \right[\times [t^n, t^{n+1}]$ on which the physical PDE will be integrated by the finite volume method.

3.2. Description of the Physical PDE Integration by the Finite Volume Method

This second step is devoted to the description of the establishment of the numerical scheme which will allow the calculation of the values of the unknown at the nodes defined by the discretization described above. Let us consider the hyperbolic conservation law with the following diffusion term:

$$\partial_t U + \partial_x (f(U)) = \varepsilon \partial_{xx} U \tag{5}$$

According to the experiences made on a number of problems tested in [14] by K. LAMIEN, L.SOME and M.OUEDRAOGO, it appears that it is necessary to reformulate the physical PDE in the form 6, to the otherwise, no results are obtained. The scheme are established with the reformulation of the conservation law which is written:

$$\partial_t U + \partial_x \left(\tilde{f}(U)\right) = 0 \tag{6}$$

where $\tilde{f}(U) = f(U) - \varepsilon \partial_x U$
Let's assume the mesh described above and then consider the integration of ??, on each elementary volume: $V_i^n =]x_{i-\frac{1}{2}} - x_{i+\frac{1}{2}}[\times[t^n, t^{n+1}]$ with $C_i^n =]x_{i-\frac{1}{2}} - x_{i+\frac{1}{2}}[$.

$$\int\int_{V_i^n} \left(\partial_t U + \partial_x \tilde{f}(U)\right) dxdt = 0 \tag{7}$$

$$\int_{C_i^n} (U(x, t^{n+1}) - U(x, t^n))dx + \int_{t^n}^{t^{n+1}} (\tilde{f}(U(x_{i+\frac{1}{2}}, t) - \tilde{f}(U(x_{i-\frac{1}{2}}, t))dt = 0 \tag{8}$$

The value of the unknown U located at the abscissa x_i and at the time t^n is noted U_i^n and defined by her average value over V_i^n:

$$U_i^n = U(x_i, t^n) = \frac{1}{Ax_i^n} \int_{C_i^n} U(x, t^n)dx \tag{9}$$

with $Ax_i^n = x_{i+1/2}^n - x_{i-1/2}^n$ $\Delta x_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$
The scheme after integration is written:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{Ax_i^n}\left[F(U_{i+\frac{1}{2}}^{n,-}, U_{i+\frac{1}{2}}^{n,+}) - F(U_{i-\frac{1}{2}}^{n,-}, U_{i-\frac{1}{2}}^{n,+})\right] \tag{10}$$

with F the numerical flow $F(U_{i+\frac{1}{2}}^{n,-}, U_{i+\frac{1}{2}}^{n,+}) = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} \tilde{f}(U(x_{i+\frac{1}{2}}, t)dt$. With the help of the flux limiters, we write:

$$U_{i+\frac{1}{2}}^{n,-} = U_i^n + \tilde{\psi}_i^n(x_{i+\frac{1}{2}} - x_i), \quad U_{i+\frac{1}{2}}^{n,+} = U_{i+1}^{n,+} + \tilde{\psi}_{i+1}^n(x_{i+1} - x_{i+\frac{1}{2}}) \tag{11}$$

The flux limiters are used to avoid possible oscillations in the final numerical scheme, especially when the solution sought is very steep. Also an off-center upstream approximation is used for approach the numerical stream, hence

$$F(U_{i+\frac{1}{2}}^{n,-}, U_{i+\frac{1}{2}}^{n,+}) = \tilde{f}(U_{i+\frac{1}{2}}^{n,-})$$

$$F(U_{i-\frac{1}{2}}^{n,-}, U_{i-\frac{1}{2}}^{n,+}) = \tilde{f}(U_{i-\frac{1}{2}}^{n,-})$$

The final scheme is written:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{Ax_i^n}\left(\tilde{f}(U_{i+\frac{1}{2}}^{n,-}) - \tilde{f}(U_{i-\frac{1}{2}}^{n,-})\right) \tag{12}$$

In order to obtain a higher accuracy in the time range, the scheme ?? is replaced by a second-order Runge Kunta scheme:

$$\begin{cases} U_i^{(1)} = U_i^n + \Delta t L(U^n) \\ U_i^{n+1} = \frac{1}{2}\left(U_i^n + U_i^{(1)} + \Delta t L(U^{(1)})\right) \end{cases} \tag{13}$$

with $L(U^n) = \frac{1}{\Delta x_i}\left(F(U_{i+\frac{1}{2}}^{n,-}, U_{i+\frac{1}{2}}^{n,+}) - F(U_{i-\frac{1}{2}}^{n,-}, U_{i-\frac{1}{2}}^{n,+})\right)$

4. NUMERICAL RESULTS

The purpose of this section is to validate the numerical solutions obtained by the method of finite volumes with a moving mesh through a series of comparisons.

4.1. Some Characterizations of the Numerical and Reference Solutions

Figures 1 and 3 show the initial conditions of the two problems, figures 2 and 4 give the numerical solutions obtained by the finite volume method with a static mesh which will be considered in the following as reference solutions.

In figures 5 and 7, we observe the trajectories of the mesh nodes. And finally on figures 6 and 8, we have the graphical numerical solutions obtained by finite volume method with a moving mesh.

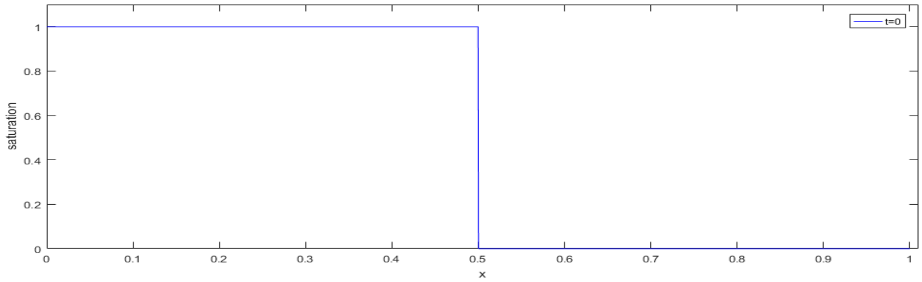

Figure1. *Initial solution to the saturation problem*

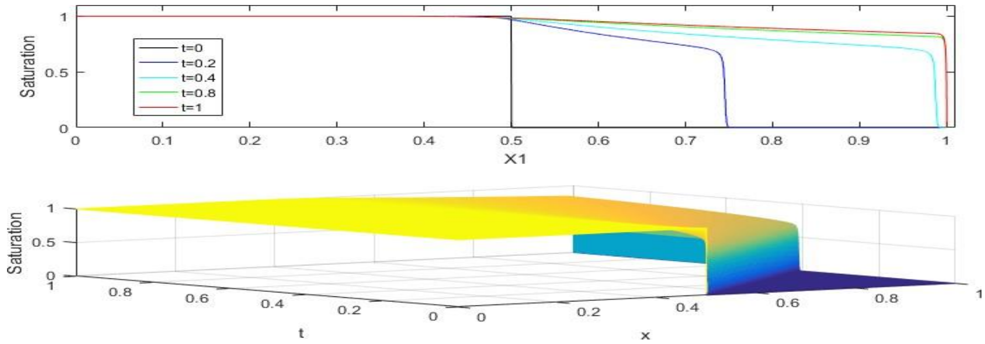

Figure2. *Solution to the saturation problem determined by the finite volume method with a staticmesh of 2000 nodes*

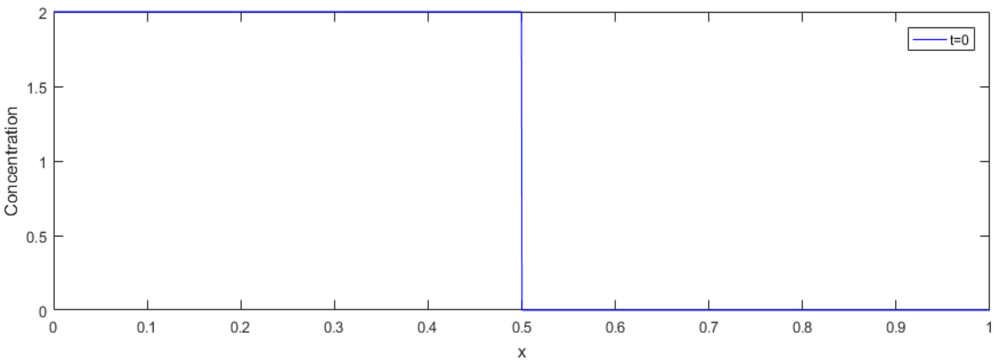

Figure3. *Initial solution to the transport-diffusion problem*

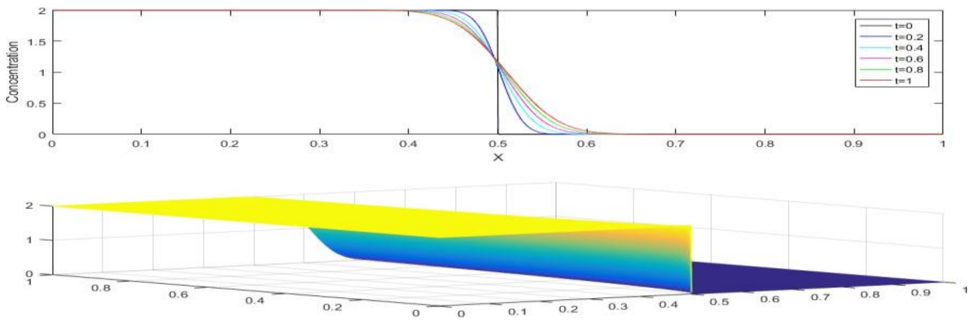

Figure4. *Solution to the transport-diffusion problem determined by the finite volume method witha static mesh of 1650 nodes.*

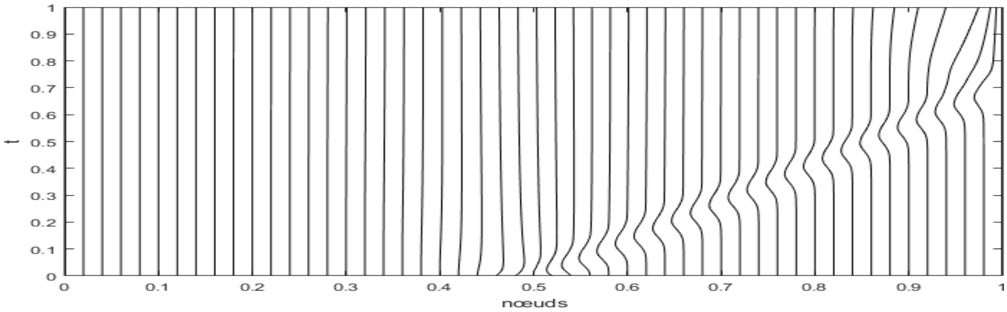

Figure5. *Saturation problem: trajectories of 51 nodes*

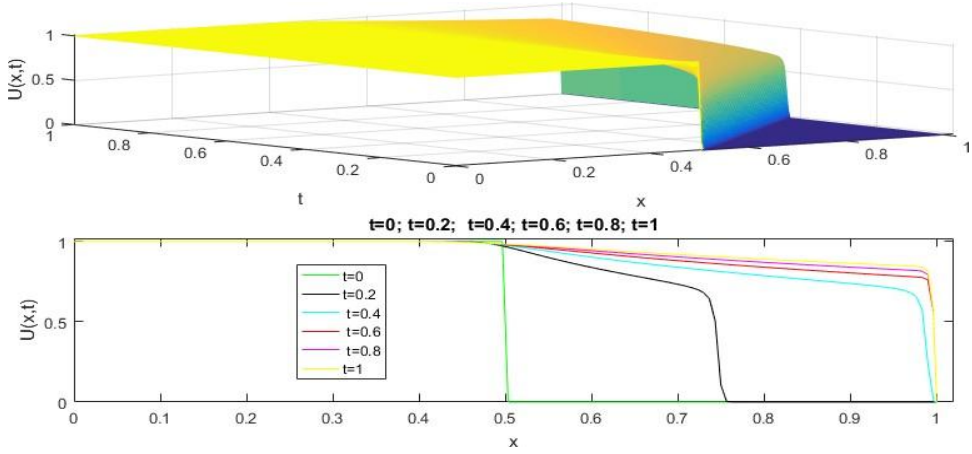

Figure6. *Solution of the saturation problem determined by the finite volume method with a movingmesh of 150 nodes.*

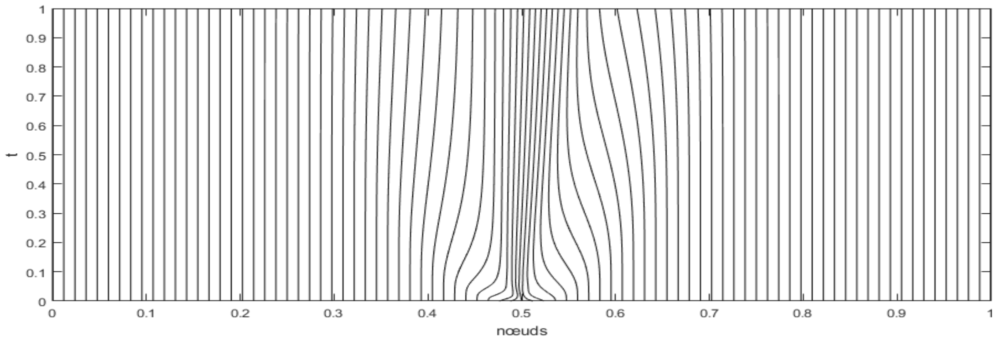

Figure7. *Transport-diffusion problem: trajectories of 51 nodes*

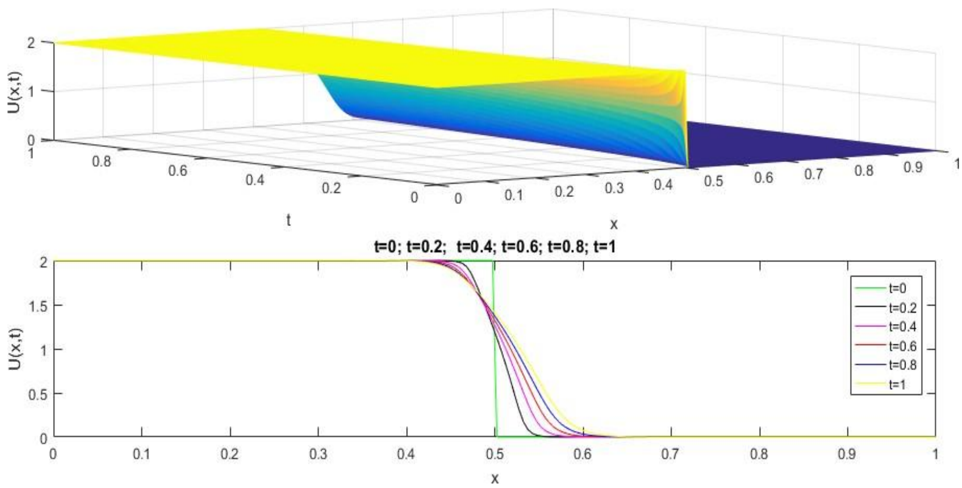

Figure8. *Solution of the transport-diffusion problem determined by the finite volume method witha moving mesh of 200 nodes.*

4.2. Validation of numerical solutions obtained by the finite volume method with a moving grid

The aim is to evaluate the numerical solutions obtained by the finite volume method with a moving mesh by comparing them to reference solutions. The reference solutions are those determined bythe finite volume method with a static mesh, which will also be validated by comparing them to those determined by finite differences method. The aim is to be able to highlighting the main advantage of a moving mesh, which is to determine with a more high accuracy the solutions rather irregular using fewer discretization nodes than the static mesh. As a reminder, the studied methodis the finite volumes method with a moving mesh, hence the choice of reference solutions.

4.2.1. Evaluation of reference solutions

This involves validating the numerical solution determined by the finite volume method with a static mesh by comparing it to the numerical solution determined by the finite difference method .

This comparison will be made through the graphical overlay both numerical solutions on the same figure, but also through an estimation of the difference between the two solutions in an table. The error is said to be estimated, since the two solutions noted U_{vf} and U_{df} respectively determined by finite volumes and finite differences do not have the same size as vectors.

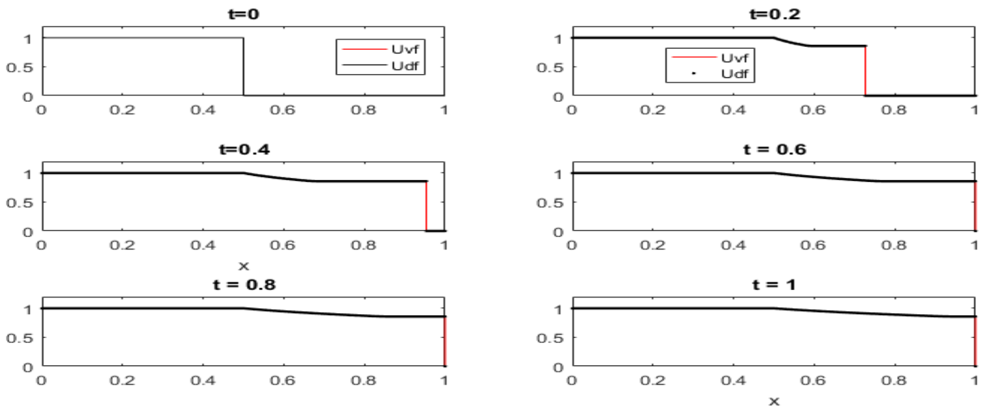

Figure9. *Saturation problem: Graphical comparison of the solutions determined by the finite volumes method and the finite difference method both with a static mesh*

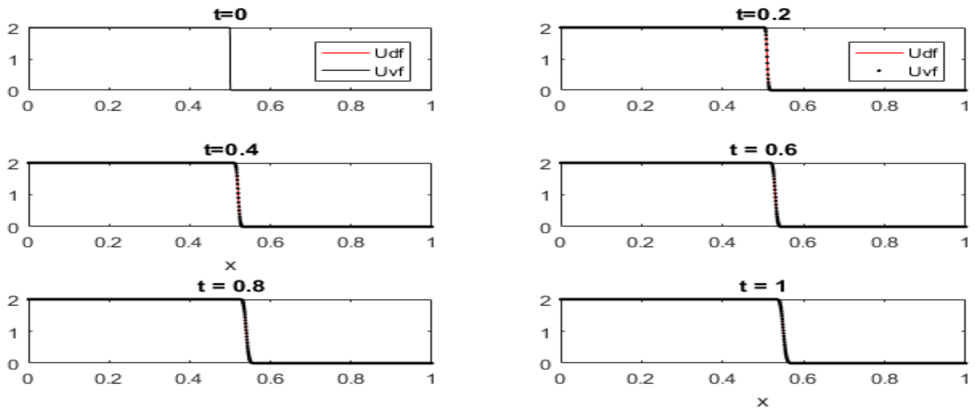

Figure10. *Transport-diffusion Problem: Graphical comparison of the solutions determined by the finite volumes method and the finite difference method both with a static mesh*

Table1. *Table of estimated error*

	Saturation problem				
Time sections	0.2	0.4	0.6	0.8	1
E_2	1.2723	1.49	0.4896	0.5277	0.55
E_∞	0.5695	0.5950	0.3107	0.3196	0.32
	Transport-diffusion problem				
E_2	0.0163	0.0274	0.0371	0.0460	0.0544
E_∞	0.0062	0.0088	0.0108	0.0124	0.0139

4.2.2. Comments

In Table 1, we have an estimate of the difference between the solution obtained by the finite differences method and that obtained by the finite volume method with a static mesh considered asa reference solution.

In figures 9 and 10, the superposition states of the solutions determined by the finite volume method denoted U_{vf} and the finite differences method denoted U_{df} , thus Table 1 show the accuracy of the finite volume method compared to the finite difference method. Remember that the values in Table1 are not significant because they are estimated values.

Indeed, U_{vf} and U_{df} are two vectors of different sizes. To obtain these values, it was necessary toapproach the larger vector size and then evaluate it at the nodes of the smaller vector size. The values in table 1 are therefore for information only. It is in this context, we set the finite volume method solution as a benchmark solution against which we will evaluate the solution noted U_{num} determined by the finite volume method this time with a moving mesh.

4.2.3. Graphical comparison of the finite volume method and adaptive moving mesh method graphical comparison

The numerical solution as said above, determined by the finite volume method with a static meshwill be considered as the reference solution to be used to evaluate the one determined by the same method with a moving mesh.

Since the estimated error is not very significant, this comparison will be made through a graphical study only. In order to appreciate the accuracy of the adaptive moving mesh method denoted M_{num} determined with few nodes (only a few hundred), we will simultaneously represent on four different figures the reference solution determined with a static mesh of 2000 nodes for the first problem and 1650 nodes for the second, as well as the adaptive moving mesh method M_{num} determined by 100, 300, 400 and 600 nodes respectively.

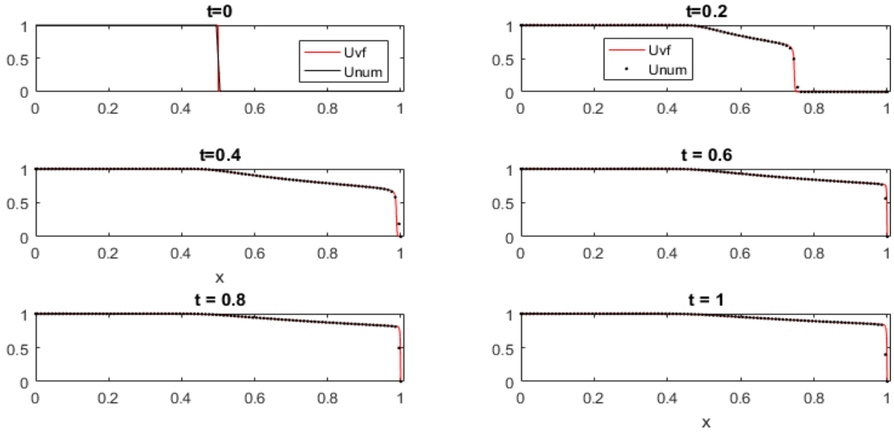

Figure11. *Saturation problem: Graphical comparison of the finite volume method (2000 nodes) and the adaptive moving mesh method (100 nodes) at the following times t = 0, t = 0.2s, t = 0.4s,t = 0.6, t = 0.8s, t = 1s*

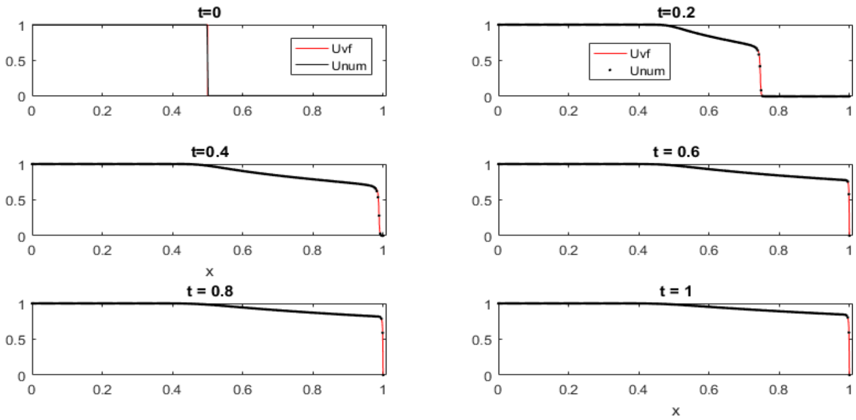

Figure12. *Saturation problem: Graphical comparison of the finite volume method (2000 nodes) and the adaptive moving mesh method (300 nodes) at the following times t = 0, t = 0.2s, t = 0.4s,*

t = 0.6, t = 0.8s, t = 1s

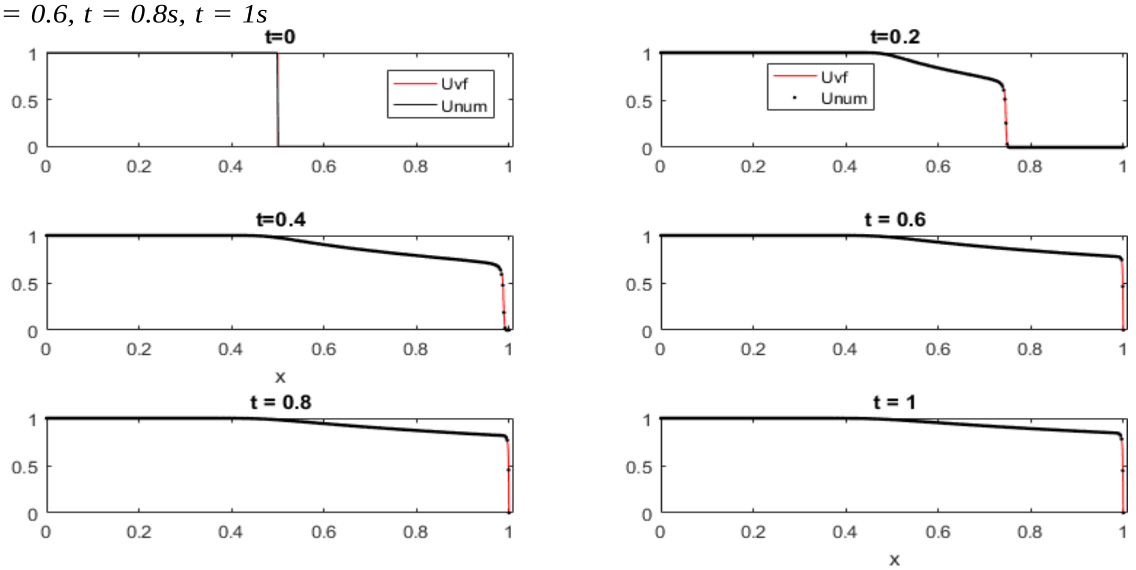

Figure13. *Saturation problem: Graphical comparison of the finite volume method (2000 nodes) and the adaptive moving mesh method (400 nodes) at the following times t = 0, t = 0.2s, t = 0.4s, t = 0.6, t = 0.8s, t = 1s*

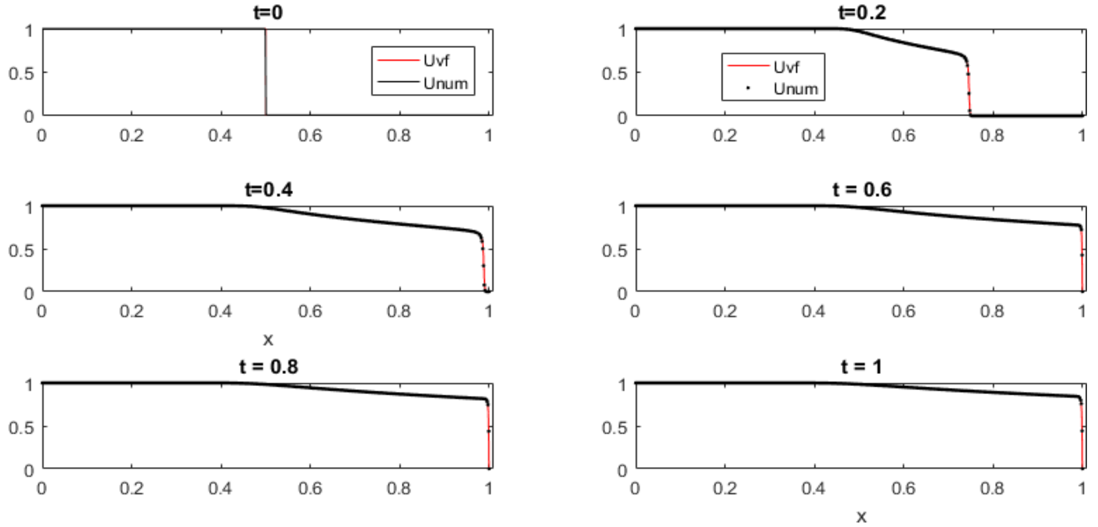

Figure14. *Saturation problem: Graphical comparison of the finite volume method (2000 nodes) and the adaptive moving mesh method (600 nodes) at the following times t = 0, t = 0.2s, t = 0.4s,t = 0.6, t = 0.8s, t = 1s*

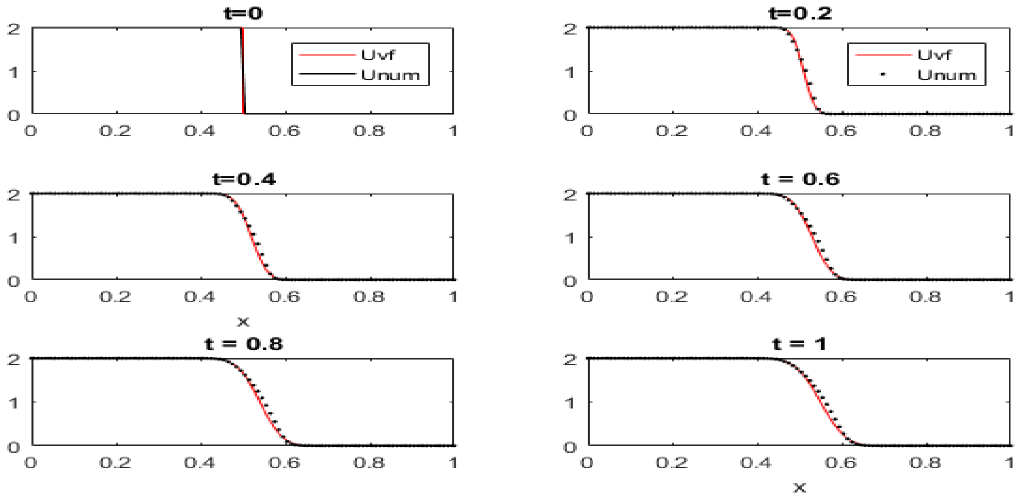

Figure15. *Transport-diffusion problem: Graphical comparison of the finite volume method (1650*

nodes) and the adaptive moving mesh method (100 nodes) at the following times t = 0, t = 0.2s,t = 0.4s, t = 0.6, t = 0.8s, t = 1s

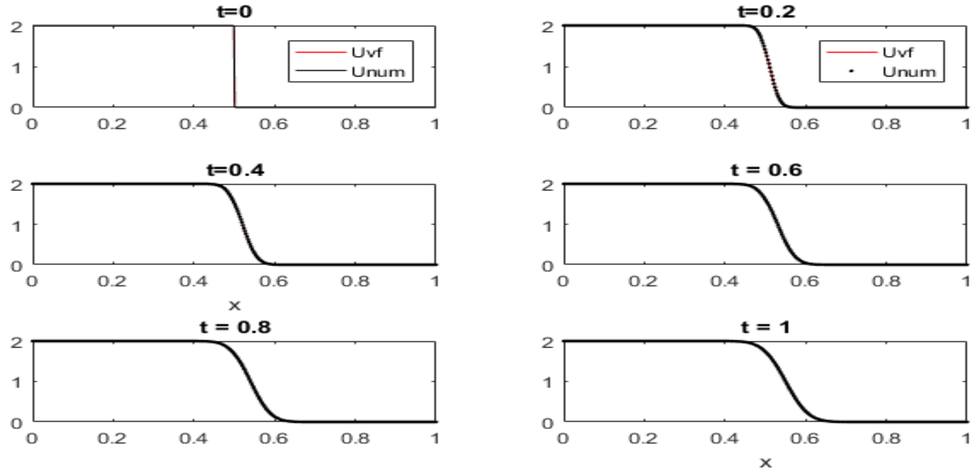

Figure16. *Transport-diffusion problem: Graphical comparison of the finite volume method (1650 nodes) and the adaptive moving mesh method (300 nodes) at the following times t = 0, t = 0.2s,t = 0.4s, t = 0.6, t = 0.8s, t = 1s*

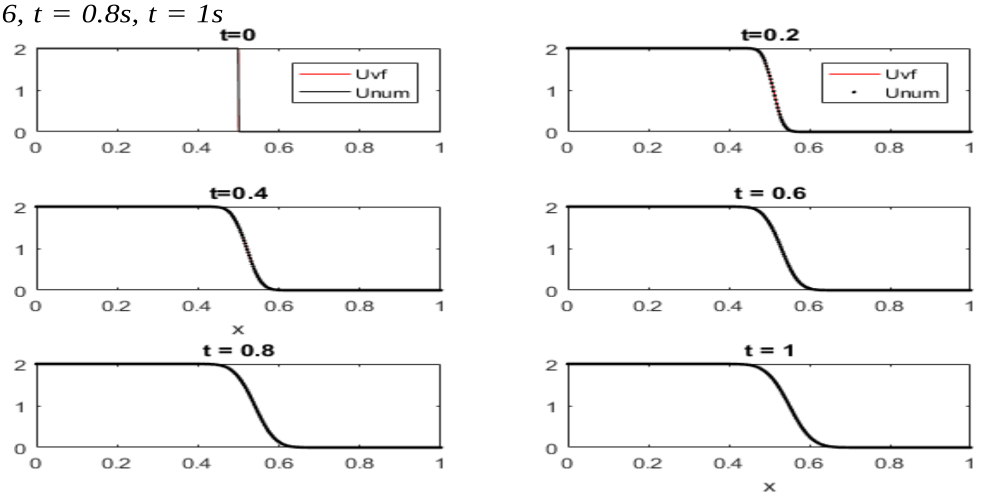

Figure17. *Transport-diffusion problem: Graphical comparison of the finite volume method (1650 nodes) and the adaptive moving mesh method (400 nodes) at the following times t = 0, t = 0.2s,t = 0.4s, t = 0.6, t = 0.8s, t = 1s*

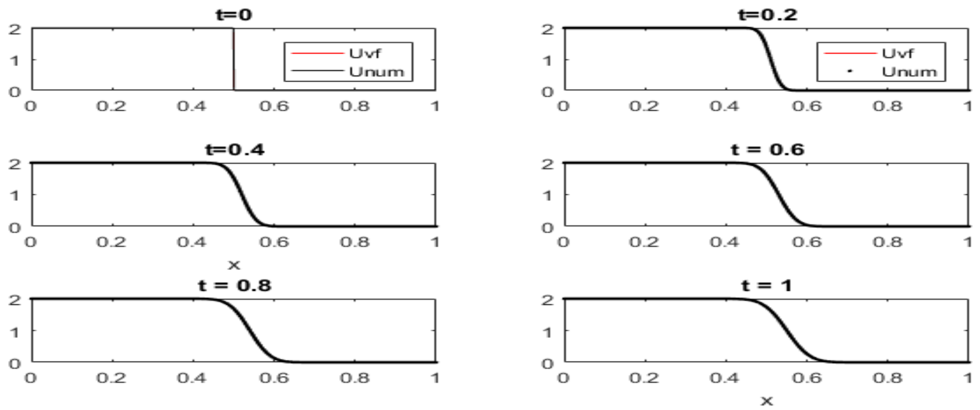

Figure18. *Transport-diffusion problem: Graphical comparison of the finite volume method (1650 nodes) and the adaptive moving mesh method (600 nodes) at the following times t = 0, t = 0.2s,t = 0.4s, t = 0.6, t = 0.8s, t = 1s*

4.2.4. Comments

Through figures 11 to 14 for the saturation problem and figures 15 to 18 for the diffusion transportproblem, we note mainly that already with 100 nodes the adaptive moving mesh method has a fairly good accuracy. We also note that this accuracy improves with increasing the number of nodes. In conclusion, we confirm that the adaptive moving mesh method offers more possibilities to obtain better results.

5. CONCLUSION

This paper is an experiment which consisted in solving by the finite volume method with a moving mesh, two hyperbolic conservation laws whose particularity is that they form Riemann problems consisting of physicals PDEs and discontinuous initial conditions. This experiment has consists in first presenting the two chosen problems, and then in a second step, in presenting the finite volume method with a moving mesh was described, before ending with the validation of the results obtained through a series of comparisons. The result obtained is well in conformity with the objectives which supported the development of the adaptive moving mesh method. These are as follows: use few nodes in the grids to obtain sufficiently precise numerical solutions.

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