International Journal of Scientific and Innovative Mathematical Research(IJSIMR)
Volume 8, Issue 3, 2020, PP 6-8
ISSN No. (Online) 2347-3142
DOI: http://dx.doi.org/10.20431/2347-3142.0803003
www.arcjournals.org

Proposing a New Lemma about Odd Numbers and a New Conjecture about a Sequence of Prime Numbers

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Abstract: *Here in this article firstly I will prove a new lemma and the lemma is, every odd number which is not a multiple of 3 and greater than 1 can be represented as* $\sqrt{2^{2+n}.3k+1}$ *where n and k are positive integers. Then I will propose a new conjecture about a sequence of prime numbers.*

Keywords: *Number theory, odd numbers, multiple of three, positive integers, prime numbers, conjecture, lemma.*

1. Introduction

In mathematics, informal logic and argument mapping, a lemma is a generally minor, proven proposition which is used as a stepping stone to a larger result. For that reason, it is also known as a ''helping theorem'' or an ''auxiliary theorem''. Here I have proven a new lemma and the lemma is, every odd number which is not a multiple of 3 and greater than 1 can be represented as $\sqrt{2^{2+n}.3k+1}$ where n and k are positive integers. To prove this lemma we have to use some theorems about odd numbers. Again, a conjecture is a conclusion or a proposition which is suspected to be true due to preliminary supporting evidence, but for which no proof or disproof has yet been found. Here I will propose a new conjecture about a sequence of prime numbers.

2. The New Lemma

The lemma is as follows,

Lemma 1: "Every odd number which is not a multiple of 3 and greater than 1 can be represented as $\sqrt{2^{2+n}.3k+1}$ where n and k are positive integers."

The theorem which is needed to prove this lemma,

Theorem 1: "The square of odd numbers minus 1 is divisible by 8."

The square of multiples of 3 minus 1 is never divisible by 3 because the square of any multiple of 3 is also a multiple of 3 and after subtracting 1 it will be an even number. This even number won't be divisible by 3. To prove the lemma we have to prove that the square of any number which is not a multiple of 3 minus 1 is divisible by 3k.

Now, we can represent every number (not the multiples of 3) by using the multiples of 3. Every number can be represented as $3k+1$ (1) or $3k+2$ (2) or $3k-1$(3)or $3k-2$ (4).

Now, squaring the numbers which are not the multiples of 3 and subtracting 1 we get,

$$(3k+1)^2-1=9k^2+6k+1-1=3k(3k+2)$$

$$(3k+2)^2-1=9k^2+12k+4-1$$

$$=3(3k^2+4k+1)$$

$$(3k-1)^2-1=9k^2-6k+1-1=3k(3k-2)$$

$$(3k-2)^2-1=9k^2-12k+4-1=3(3k^2-4k+1)$$

Here we can see that $3k(3k+2)$ and $3k(3k-2)$are divisible by 3k. Again, $3(3k^2+4k+1)$ and $3(3k^2-4k+1)$ are divisible by 3. In this case k = 1. So we can say the square of any number which is not a multiple of 3 minus 1 is divisible by 3k. Again, The square of odd numbers minus 1 is divisible by 8 or 2^3. That's why, the square of every odd number which is not a multiple of 3 minus 1 is divisible by 3 and 2^3. The square of 1 minus 1 is 0 and 0 is divisible by every number. So we have to take all the odd numbers greater than 1. Now look at the table below,

Odd Numbers	The square of the odd numbers minus 1	Divisible by
3	$3^2 - 1 = 8$	2^3
7	$7^2 - 1 = 48$	2^4
23	$23^2 - 1 = 528$	2^4
31	$31^2 - 1 = 960$	2^6

Now we can write,

The square of odd numbers minus 1 is divisible by at least 8. But it can be divisible by 2^4, 2^5, 2^6 etc. So we can write, The square of odd numbers minus 1 is divisible by 2^{2+n} where n is any positive integer.

Now we get,

(1) The square of any number which is not a multiple of 3 minus 1 is divisible by 3k.

(2) The square of odd numbers minus 1 is divisible by 2^{2+n} where n is any positive integer.

Combining (1) and (2) we can write, the square of any odd number which is not a multiple of 3 minus 1 is divisible by $2^{2+n}.3k$. So we can represent the odd number which is not a multiple of 3 and greater than 1 as $\sqrt{2^{2+n}.3k+1}$. For example $5 = \sqrt{2^{2+1}.3.1+1}$, $7 = \sqrt{2^{2+2}.3.1+1}$ etc.

3. THE CONJECTURE

The conjecture is as follows,

"Suppose, set S = { 10, 20, 30, 40,…..}. If there are three or four prime numbers between n_a and (n_a+ 10) and also between n_b and (n_b+ 10) where n_a,n_b ∈ S and $n_a > n_b$ or $n_a < n_b$,then the difference between n_a and n_b ($n_a \sim n_b$) will be a multiple of 30."

For example:

There are four prime numbers (11, 13, 17, 19) between 10 and (10+10) or 20 and three prime numbers (41, 43, 47) between 40 and (40+10) or 50. Here in this case n_a= 10 and n_b= 40. So, the difference between n_a and n_b is 30 which is a multiple of 30.

Again, There are three prime numbers (71, 73, 79) between 70 and (70+10) or 80, three prime numbers (223, 227, 229) between 220 and (220+10) or 230 and three prime numbers (311, 313, 317) between 310 and (310+10) or 320. So, the difference between 70 and 220 is 150 or (30 × 5) which is a multiple of 30. Again the difference between 220 and 310 is 90 and the difference between 70 and 310 is 240. Here 90 and 240 are also the multiples of 30.Here we can give thousands of example of this conjecture because we have tried this by all the prime numbers from 1 to 100000. But we don't have any proof of this conjecture.

4. CONCLUSION

The lemma is interesting and can be very important for number theory. The conjecture is still unsolved. Anybody can prove or disprove it.

ACKNOWLEDGEMENTS

I should Thanks MD. Shah Alam and MST. Sabina Yesmin for providing me with the help materials.

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Citation: *Salman Mahmud, Proposing a New Lemma about Odd Numbers and a New Conjecture about a Sequence of Prime Numbers, International Journal of Scientific and Innovative Mathematical Research (IJSIMR), vol. 8, no. 3, pp. 6-8, 2020. Available : DOI: http://dx.doi.org/10.20431/2347-3142.0803003*