



A Comparison of Four Different Garch Models in Modelling Different Stocks Selected from Nigerian Stock Exchange

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Abstract: This study compared four different Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models in modelling different stocks selected from Nigerian stock exchange using daily data from the period of October 2015 to July 2021 and the data for each stock was shown to be asymmetric. The analysis began with pre-diagnostic test for estimating GARCH model that is testing for clustering volatility and ARCH effect in the residual. The properties of the time series variables were examined and tested for stationarity using the Augmented Dickey-Fuller (ADF) unit root test. The test revealed that all the stocks; Total, Dangote, Unilever and Nestle were stationary at first difference $I(1)$. The test for the co-integration carried out provided clear evidence of co-integration hence showing long-run relationship. The best volatility model was determined after comparison among the estimated model (GARCH, I-GARCH, E-GARCH and FIGARCH) and the results shows that fractional integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) provided the best fit for (Dangote stock, Total stock and Unilever stock) having the lowest Akaike information criterion (AIC) and Bayes information criterion (BIC) values while standard GARCH provide a best fit for Nestle stock having the lowest AIC and BIC. Hence we can conclude that for long run relationships that FIGARCH is most recommended.

Keywords: Co-integration, Volatility, GARCH, IGARCH, EGARCH and FIGARCH

1. INTRODUCTION

Monetary resource returns are generally portrayed by unpredictability grouping, that is to say, broadened times of "vicious" or high market instability followed by a time of high unpredictability, and "quiet" or low market unpredictability followed by time of low unpredictability. This is referred to as volatility in Economics; Volatility is defined as statistical measure of the dispersion of returns for say a market index which is a monetary resource return. Furthermore, monetary time series will generally show negative skewness, abundance kurtosis, and worldly tirelessness in restrictive difference Andersen et al, [1]. Other than these, monetary resources returns are seen to frequently have thicker tails than anticipated under ordinarieness. A few investigations recommend that these tails may be so thick as to have come from a Cauchy conveyance, or different disseminations with limitless minutes Mandelbrot, [2]. Given that data is collected regularly and is organized by time, one of the most remarkable methods for analyzing financial transaction is time series analysis. Time series data in finance could include data on daily conversion standards, daily offer costs, daily share files, etc. Information from time series may be fixed or unfixed. If the mean and difference of a succession or series do not precisely alter, it is fixed or strictly fixed. An instability model is required by this stationarity. Financial managers therefore place a great deal of emphasis on choosing the right unpredictability model to identify variations in stock returns. Financial backers are helped even more in this way by reliable unpredictability models of resource returns when making wagering decisions and portfolio adjustments. There are numerous studies and various models about the unpredictability of financial information. Financial data have demonstrated that a few factors, such as an excess of kurtosis, negative skewness, and ephemeral persistentness in contingent developments, are involved in the restricted diffusion of high recurrent returns. To oblige them, econometricians have created devices at demonstrating and estimating unpredictability. Among these models, the Autoregressive Regressive Conditional Heteroskedasticity (Curve) model proposed by Engle [3] and its expansion, the General Autoregressive Conditional Heteroskedasticity (GARCH) model by Bollerslev [4], were viewed as the main models brought into the writing and they have become extremely famous in that

they empower an examiner to gauge the difference of a series at a specific moment. The family of GARCH models namely; Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH), Integrated Generalized Autoregressive Conditional Heteroskedasticity (I-GARCH) and Fractional Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) were applied in this work and comparison of four different garch models in modelling different stocks selected from Nigerian stock exchange was done. The stocks considered are: Total, Dangote, Unilever and Nestle stocks.

2. LITERATURE REVIEW

GARCH models as a rule show a high constancy of the contingent change. The Curve and the GARCH models catches instability bunching and leptokurtosis. In circumstances where their circulations are symmetric, they neglect to demonstrate the influence impact. To resolve this issue, numerous nonlinear expansions of GARCH have been proposed, like the outstanding GARCH (EGARCH), GJR-GARCH, PGARCH and TGARCH. In this way, the evaluations of a GARCH model in the constancy boundary might experience the ill effects of a significant vertical predisposition. Hence, models in which the boundaries are permitted to change over the long haul might be more fitting for demonstrating unpredictability. Baillie et al [5] proposed a new class of fractional integrated generalized autoregressive conditional heteroscedastic model (FIGARCH) which are consistent with long run dependencies in absolute and square assets returns. The ARFIMA and FIGARCH models may be sensitive to the specification used to represent the short-run dynamics, whereas this estimator allows for rather generic forms of short-run dynamics Künsch, [6], like the tests suggested by Shimotsu [7]. Also the Adaptive-FIGARCH of Baillie and Morana [8], the two-step approach of Morana and Beltratti [9], and the procedure devised by Ohanissian et al. [10] are just a few of the different ways that may be used to account for extended memory. If underlying data generation process is unknown, structural breakdowns do not need to be identified. By providing precise volatility estimates for the Nigerian market, regulators, the government, traders, and investors are given the chance to create better regulations and select the right financial investments. "However, if structural breakdowns are neglected, the precision of interval forecasting and the accuracy of volatility estimations are both weakened". Emenogu et al [11] investigated the volatility in daily stock returns for Total Nigeria Plc using nine variants of GARCH models. Okoye et al. [12] empirically examined the interrelationship between the construction sector, oil prices and gross domestic product (GDP) in Nigeria, finding short-run linear relationships among these macroeconomic variables. They argued that neither the construction sector nor oil prices directly influence the aggregate economy. The issue of cointegration occurs mostly in financial analysis; Akinlo [13] examined the relationship between oil prices and the stock market in Nigeria using the Vector Error Correction Model approach. The study revealed that oil prices, the exchange rate and stock market development are cointegrated, while the price of oil has a temporary positive impact on stock market growth in Nigeria

3. MATERIALS AND METHODS

3.1. Research Design

This work used quasi-experimental design from a broader perspective of classical experimental design as a logical-proof approach. The data was obtained is daily data from 20th of October, 2015 to 28th of July, 2021 extracted from the Central Bank of Nigerian bulletin. The selected stocks are stock market exchange index of Dangote, Total, Nestle and Unilever companies in Nigeria.

3.2. Sample and Sampling Technique

According to Nigerian Exchange Group (2021), the Nigerian Stock Exchange has 328 stocks, and because it is impossible to deal with all of them, systematic random selection procedures were employed to pick the stocks used. Four stocks were chosen as the sample unit since the the population size (N) was known, hence the sampling interval was simply divided by "N." *Mathematically*, $K=N/n$

Where

K= Sampling interval

N=Entire Population (328)

n=Sample Size (4)

Therefore, $K = \frac{328}{4} = 82$ which gives approximately 4 stocks. Therefore every 82nd market stock was chosen from the list for the study, while the first was selected by chance. Based on this, Total, Dangote, Unilever and Nestle stocks were chosen. Conditional variance models are fitted to daily stock returns that are continually compounded.,

$$y_t: y_t = 100(\ln k_t - \ln k_{t-1}) \quad (1)$$

Where k_t = current period of stock market exchange, k_{t-1} = previous period stock market exchange, y_t = current period stock returns (stock market exchange -RT).

3.3. Models of Volatility

The mean and variance equations of the Family of ARCH Models (Autoregressive Conditional Heteroskedasticity) are two distinct requirements for each ARCH or ARCH family model. Engle asserts that conditional heteroskedasticity in a return series can be modeled using the ARCH model with the mean equation of the following form:

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (2)$$

Such that $\varepsilon_t = \varphi_t \sigma_t$

3.2.1 The unconditional kurtosis of ARCH (1)

Suppose the innovations are normal, then

$$E \left(\frac{\alpha_t^4}{E_{t-1}} \right) = 3 \left[E \left(\frac{\alpha_t^2}{E_{t-1}} \right) \right]^2 \quad (3)$$

$$= 3 (\alpha_0 + \alpha_1 \alpha_{t-1}^2)^2$$

It follows that

$$E \alpha_t^4 = \frac{3\alpha_0^2(1+\alpha_1)}{[(1-\alpha_1)(1-3\alpha_1^2)]} \quad (4)$$

and

$$\frac{E \alpha_t^4}{(E \alpha_t^2)^2} = 3(1 - \alpha_1^2) / (1 - 3\alpha_1^2) > 3 \quad (5)$$

This shows that the tail distribution of a_t is heavier than that of a normal distribution.

3.2.2 Generalized ARCH (GARCh) Model

There are many univariate ARCH-type models for heteroskedasticity since Engle [3]. The ARCH idea included clustering and shock persistence. Different models differ based on the facts they can address. ARCH inspires numerous conditional heteroskedastic models. The most prominent volatility models, GARCh (q, p) and EGARCh (q, p), use "conditional heteroskedasticity" to model data volatility (exponential-GARCh). According to Rossi [14], the asymmetric power ARCH model proposed by Ding et al. [15] forms the basis for deriving the GARCh family models. The conditional variance for GARCh (p, q) model is expressed generally as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (6)$$

Where:

σ_t^2 is the conditional variance at time t

q is the order of ARCH component model

$\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_q$ are the parameters of the ARCH component model

p is the order of GARCh component model

$\beta_1, \beta_2, \beta_3, \dots, \beta_p$ are the parameters of GARCh component model

ε_t^2 , is the disturbance term.

The reduced form of equation 3.6 is the GARCH (1, 1) represented as:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \quad (7)$$

The three parameters (β_0 , β_1 and β_2) are nonnegative and $\beta_1 + \beta_2 < 1$ to achieve stationarity.

3.2.3 E-GARCH model

The exponential GARCH (EGARCH) model was proposed by Nelson [16] to overcome some weaknesses of the GARCH model in handling financial time series, as pointed out by Enocksson and Skoog [17]. It captures the leverage of GARCH model and its given as;

$$\ln \sigma_{t+1}^2 = \omega + \alpha(\varphi R_t + \gamma[|R_t| - E|R_t|]) + \beta \ln \sigma_t^2 \quad (8)$$

Which displays the usual leverage effect if $\alpha\varphi < 0$. The EGARCH model has the advantage that the logarithmic specification ensures that variance is always positive, but it has the disadvantage that the future expected variance beyond one period cannot be calculated analytically. A more general exponential GARCH, or EGARCH model is;

$$\log(\sigma_t) = \alpha_0 + \sum_{i=1}^q \alpha_i g(\varepsilon_{t-1}) + \sum_{i=1}^p \beta_i \log(\sigma_{t-1}) \quad (9)$$

Where:

σ_t is the conditional standard deviation at time t.

q is the order of ARCH component model

$\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_q$ are the parameters of the ARCH component model

p is the order of GARCH component model

$\beta_1, \beta_2, \beta_3, \dots, \beta_p$ are the parameters of GARCH component model

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma\{|\varepsilon_t| - E(|\varepsilon_t|)\} \quad (10)$$

3.2.4. Integrated GARCH model (IGARCH)

Integrated GARCH (IGARCH) models are unit-root GARCH models. The I-GARCH(1, 1) model is specified in Tsay [18] as;

$$\vartheta_t = \sigma_t e_t;$$

$$\sigma_t^2 = \omega_0 + \beta_1 \sigma_t^2 + (1 - \beta_1) \vartheta_{t-1}^2 \quad (11)$$

Where $0 < \beta_1 < 1$, e is normally distributed with mean 0 and variance 1. The IGARCH processes are either non-stationary or have an infinite variance.

3.2.5. Fractional Integrated Generalized Autoregressive Conditional Heteroskedastic (FIGARCH) Model

The generalized specification for the conditional variance using FIGARCH (p, d, q) is given as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^q \gamma_i I_{t-1} \varepsilon_{t-1}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (12)$$

Where $I_{t-1} = 1$, if $\varepsilon_{t-1}^2 < 0$ and 0 otherwise

However, the first order representation is of FIGARCH (p, d, q) is

$$\sigma_t = \beta_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (13)$$

3.4. Co-integration test

This test is given as

$$Y_t = A_t Y_{t-1} + \dots + A_p Y_{t-p} + B_y + e_t \quad (14)$$

Where;

Y_t represents the dimensional vector of non-stationary I(I) variable,

$Y = y$ - dimensional vector of deterministic variable and e_t stochastic error residual.

So, the following is the premise for the Johansen co-integration test:

H_0 : The variables don't integrate together.

H_1 : The variables are integrated together.

The Johansen [19] approach was employed in this investigation, allowing for the estimation and testing of many co-integration relationships in a single phase. The co-integration test illustrates how the variables are related over the long term.

3.5. Unit Root Test

Works based on time series assumed that the series were stationary. However, in reality, not all economic variables are stationary in their respective states. Some variables are non-stationary, which means that their mean, variance, and covariance do not remain constant over the course of time. In other words, these variables exhibit non-constant behaviour over the course of time. It is possible for a trend to be either stochastic or deterministic; however, if the trend is totally predictive, then it is no longer referred to as a variable and is instead referred to as a determinist. A non-stationary variable is one that does not have a trend. The study used the Augmented Dickey Fuller to find out if there was a unit root. The researcher provided the following equation for the ADF method of unit root test:

$$Y_t = \beta Y_t - 1 + \mu \tag{15}$$

Where Y_t will be stationary if the estimated value of β is less than 1 and y will not be stationary of the estimated value if β is more than or equal to 1. The null and alternative hypotheses for testing the existence of unit root in the variable Y_t were:

$$H_0: \beta = 0 \quad \text{vs} \quad H_1: \beta < 0$$

Typically, the unit root is calculated on individual variables and expressed in one of three ways, as illustrated below:

$$\Delta\mu_i = \gamma_0 + \gamma_1 t + \gamma_2 \mu_i - 1 + \beta \sum_{i=1}^3 \Delta\mu_i - 1 + e_{3i} \tag{16}$$

$$\Delta\mu_i = \beta_0 + \beta_1 \mu_i - 1 + \beta \sum_{i=1}^3 \Delta\mu_i - 1 + e_{2i} \tag{17}$$

$$\Delta\mu_i = \beta \mu_i - 1 + \beta \sum_{i=1}^3 \Delta\mu_i - 1 + e_{1i} \tag{18}$$

Where; μ_i represents the variables used for the unit root test.

3.6. Pre Diagnostic Tests

We can run a GARCH model only if we can fulfill the following conditions:

a. Clustering volatility in the residual: This test is run to see if spikes in volatility are followed by spikes in volatility and spikes in volatility are followed by spikes in volatility for an extended period of time. If this criterion is met, it is likely that the residual (error term) is conditionally heteroscedastic and that the ARCH and ARCH models can be used to express it.

b. ARCH effect: The second (2nd) requirement for assessing if the ARCH model is appropriate is the ARCH effect. We reject H_0 and accept H_1 if the probability value of the chi-square is less than 5%, indicating that the ARCH model is appropriate.

3.7. Information Criteria

There are a number of information criteria that can be used to establish the order p of an AR process. The basis for all information requirements is likelihood. This work employed the Akaike information criterion (AIC) and the Schwarz-Bayesian information criterion (BIC).

4. RESULTS AND DISCUSSIONS

4.1. Nature of Selected Stocks from Nigerian Stock Exchange

Table1. Descriptive Analysis of the four Variables

Variable	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
Total	172.4	160.0	52.6	0.6	-0.1	14.31155	0.000780
Dangote	187.7	183.0	35.2	0.2	-0.8	14.03866	0.000894
Unilever	36.0	37	13.7	-0.2	-0.6	8.751690	0.012578
Nestle	1085.4	1070.0	290.7624	0.053434	-1.19237	12.24581	0.0000

Based on the results of the descriptive statistics of the data, the mean of the four stocks are 172.4, 187.7, 36, and 1085.4 respectively. The values of the kurtosis are -0.1, -0.8, -0.6, and -1.19237

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respectively, which are less than 3 hence the data are platykurtic which means they exhibit lighter tails than normal distribution (less in tails). Also, the values of the skewness are 0.6, 0.2, -0.6 and 0.053434 respectively which are not equal to zero (i.e. standard normal skewness) are said to be skewed to the right.

Volatility of selected stocks from Nigerian Stock Exchange

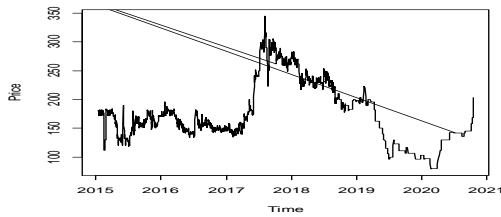


FIG1. Volatility plot of Total stock index



FIG2. Volatility plot of Dangote stock index

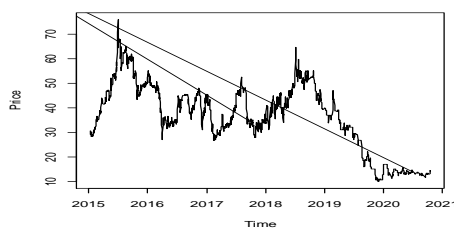


FIG3. Volatility plot of Unilever stock index

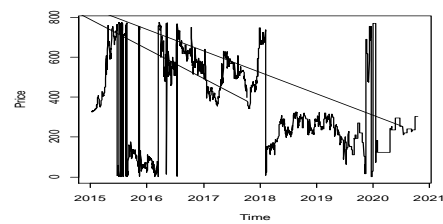


FIG4. Volatility plot of Nestle stock index

Figure 1, 2, 3 and 4 shows the non-stationarity of the volatility plots at level and reveals the need for differencing.

Table2. Johansen Co-integration Test Results

	Trace statistics	10pct	5pct	1pct	Eigenvalues (lambda)
$r \leq 3$	1.61	6.50	8.18	11.65	0.0110736855
$r \leq 2$	5.51	15.66	17.95	23.52	0.0052205727
$r \leq 1$	16.54	28.71	31.52	37.22	0.0018491757
$r = 0$	40.00	45.23	48.28	55.43	0.0007631221

The null hypothesis is rejected if the Trace statistics or Max-Eigen statistic is higher than the 0.05 critical value. Based on the result the null hypothesis of no cointegrating equation is rejected at the 5% level. Hence, it is concluded that a long-run relationship exist among the four variables.

Table3. Unit root test of stocks at level

Variables	Dickey-Fuller	Lag order	P-Value	Status
Total	-2.3418	0	0.4336	Not stationary
Dangote	-2.6533	0	0.3018	Not stationary
Unilever	-3.1295	0	0.1002	Not stationary
Nestle	-9.8792	0	0.11	Not stationary

Table 3 presents results of the Augmented Dickey Fuller (ADF) unit root test of selected stocks. The result indicates that none of the variables are stationary at their level form hence we proceed to difference them

Table4. Unit root test of stocks after first difference

Variables	Dickey-Fuller	Lag order	P-Value	Status
Total	-35.46	1	0.01**	Stationary
Dangote	-32.667	1	0.01**	Stationary
Unilever	-33.848	1	0.01**	Stationary
Nestle	-43.514	1	0.01**	Stationary

Footnote: **= Sig. at 5%

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Table 4 presents results of the Augmented Dickey Fuller (ADF) unit root test of selected stocks after the first difference. The result indicates that the entire stocks index became stationary after the first difference.

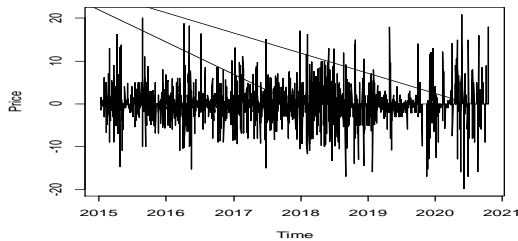


FIG5. Volatility plot of Total stock index

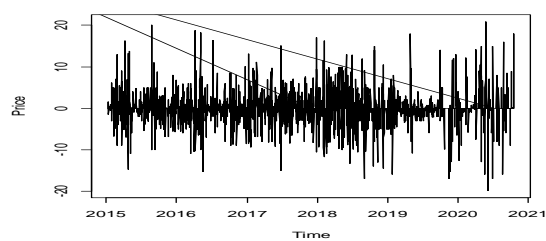


FIG6. Volatility plot of Dangote stock index

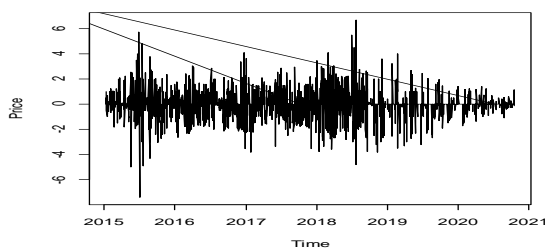


FIG7. Volatility plot of Unilever stock index

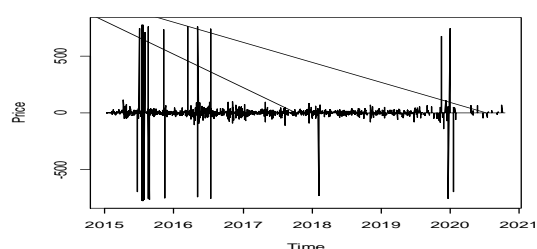


FIG8. Volatility plot of Unilever stock index

Figure 5, 6, 7, 8 shows the stationarity of series at first difference.

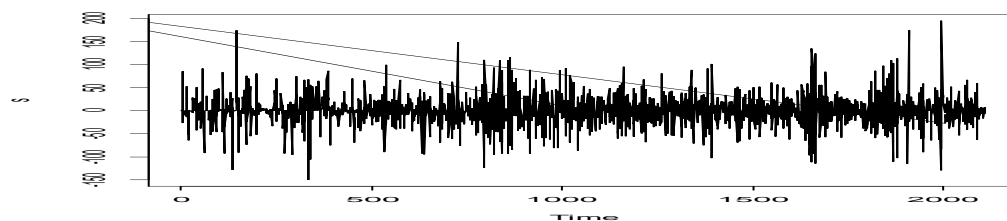


Figure9. Plot of the stationary Series of the stock market index (Total stock, Unilever stock, Dangote stock and Nestle stock)

Table5. Heteroskedasticity (ARCH Effect) Test

Chi-squared	Df	p-value
8382.3	1	2.2e-16**

Table 5 shows that the P-value associated with the chi-squared is less than 0.05 hence, we reject the null hypothesis of that there is no ARCH effect in the residual.

Table6. Results of the Mean Equation

Variables	Coefficient	Standard error	Z-statistics	Prob
Constant	99.68991	420.8161	0.236897	2.2e-16**
L(ehatsq)	0.810460	0.005882	137.79	2.2e-16**

The mean equation as presented in Table 6 above reveals that the impact of lagged stock indices is significant at 5% level of significance. This implies that previous quarter value of stock indices can influence the current quarter values of the variable. The coefficient is 0.810460 implies that there seems to be a very long delay for share prices to return to its long run position after any shock. Thus, stock index shocks are seen to be persistent over time.

Table7. Model Summary Table to determine the best forecasting volatility model

	STOCKS			
Models	TOTAL	DANGOTE	UNILEVER	NESTLE

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	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Garch	9.453	9.467	9.0898	9.0979	7.0647	7.0728	1.2146	1.2280
fiGarch	9.215	9.232	9.0375	9.0482	7.0300	7.0407	4.4077	4.4238
eGarch	9.644	9.655	9.0685	9.0792	7.0361	7.0469	7.0361	7.0469
iGarch	9.548	9.553	9.0850	9.0903	7.0310	7.0417	4.4058	4.4166

Table 7 above shows that FIGARCH performs better for Total stocks, Dangote stocks and Unilever, while GARCH performs better for Nestle.

Discussions were based on the results of the Generalized Autoregressive Conditional Heteroskedasticity (ARCH) model (1,1) and its family, the unit root tests, the Johansen co-integration test, and the results of the pre- and post-estimation diagnostic tests.

The volatility of the Total, Dangote, Unilever, and Nestle stock market index is shown in Figures 1, 2, 3 and 4. After the data have been ordered one difference at a time, or order 1, the volatility of stocks exhibits a stationarity process in the price series, these are shown in figures 5, 6, 7 and 8. Table 1 provides descriptive summary of the stock. The average for Total, Dangote, Unilever, and Nestle stock market indexes are 172.4, 187.7, 36.0, and 1085.4 values points respectively. The table reveals that all the variables have positive mean return values. The relative standard deviations of the stock market indexes for Total, Dangote, Unilever, and Nestle are 52.6, 35.2, 13.7, and 290.8 which are far below their mean values, demonstrating that their prices have been consistently unstable over time. All the variables deviate from a normal distribution, according to the P-values associated with the Jarque-Bera statistics hence asymmetric. The skewness and kurtosis coefficients of the variables provide evidence for their non-normality. The unit root test showed non-stationarity, hence it warranted differencing, differencing order 1 showed stationarity.

A test for co integration determines whether there are long-term links between the data and helps to avoid misleading regression situations. The results show presence of co-integration hence evidence of long run relationship.

Prior to estimating the ARCH model, its applicability in the analysis was tested by determining whether the residual exhibits clustering volatility and by evaluating the ARCH effect.

As a precondition for estimation of GARCH model, a test for its suitability in the analysis was conducted by estimating whether there is clustering volatility in the residual, and testing for the ARCH effect. Table 5 shows the result of the test for ARCH effect. This test presents that there is an ARCH effect in our data since the p-value of the chi-square is less than 5% with chi-square value of 8382.3. From the foregoing, there is clustering volatility in the residual and ARCH effect which implies that chosen GARCH (1.1) model is suitable for the analysis. The study therefore, proceeded to estimate the GARCH model as follows. The presence of ARCH effect with other established stylized fact of this series gave credence to the estimation of GARCH family models: the univariate conditional volatility model, namely the GARCH model, EGARCH model, the integrated model; IGARCH and fractionally integrated class of model and the FIGARCH model to determine the best volatility model.

5. CONCLUSION

Conclusively, the fractionally integrated models, namely FIGARCH (1, d, 1) performed significantly better than traditional conditional volatility models, such as GARCH (1, 1) and EGARCH (1, 1), and IGARCH (1, 1) for modelling market stock exchange futures returns on Total, Dangote and Unilever while GARCH (1,1) performed best for Nestle PLC.

REFERENCES

- [1]. Andersen, T. G., Bollerslev, T., Diebold F. X., and Ebens, X., 2001. The Distribution Of Realized Stock Return Volatility, *Journal of Financial Economics*, 61(1), 43-76, doi:10.1016/S0304-405X(01)00055-1.
- [2]. Mandelbrot B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business*, 36(4):394-419.
- [3]. Engle, R.F. (1982), Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 55, 391-407.
- [4]. Bollerslev, T. (1996), Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics*, 31, 307-327

- [5]. Baillie, R.T., Bollerslev T. and Mikkelsen H. O. (1996). Fractional Integrated Generalized Autoregressive Conditional Heteroscedastic, *Journal of Econometrics*, **74**, 3-30.
- [6]. Kunsch, H.R. (1997). Statistical aspects of self-similar process. Proceedings of the First World Congress of the Bernoulli Society, Vol. 1, pp. 67-74. Kwon, C.S. and Shin T.S., (1999). Co-integration Vectors of Economic Variables and Stock Market Returns. *Global Financial Journal*, 10: 71-81
- [7]. Shimotsu, K. (2010). Exact local whittle estimation of fractional integration with unknown mean and time trend. *Econometric Theory* 26, 501–540.
- [8]. Baillie, R.T. & Morana, C. (2009). Modeling Long Memory and Structural Breaks in Conditional Variances: an Adaptive FIGARCH Approach.
- [9]. Morana, C. and Beltratti, A. (2004). Structural change and long-range dependence in volatility of exchange rates: either, neither or both? *Journal of Empirical Finance*, Vol. 11, pp.629-658.
- [10]. Ohanissian, A., Russel, J. R. & Tsay, R.S. (2005). True or Spurious Long Memory? A New Test.
- [11]. Emenogu N. G., Adenomom M. O. and Nweze N. O. (2020). On the volatility of daily stock returns of Total Nigeria Plc: evidence from GARCH models, value-at-risk and backtesting. *Financial Innovation* (2020) 6:18. <https://doi.org/10.1186/s40854-020-00178-1>
- [12]. Okoye P.U., Mbakwe C. C. and Igbo E. N. (2018). Modeling the construction sector and oil prices toward the growth of the Nigerian economy: an econometric approach. *Economies* 6(16):1–19
- [13]. Akinlo O. O. (2014). Oil Price and stock market: empirical evidence from Nigeria. *European Journal of Sustainable Dev* 3(2):33-40
- [14]. Rossi E (2004) Lecture notes on GARCH models. University of Pavia, March.
- [15]. Ding Z, Granger CWJ, Engle RF (1993) A long memory property of stock market returns and a new model. *J Empir Financ* 1:83–106.
- [16]. Nelson D (1991) Conditional heteroskedasticity in asset pricing: a new approach. *Econometrica* 59:347–370
- [17]. Enocksson D, Skoog J (2012) Evaluating VaR (Value-at-Risk): with the ARCH/GARCH class models via. European Union: Lambert Academic Publishing (LAP)
- [18]. Tsay RS (2005) Analysis of financial time series, 2nd edn. Wiley, New Jersey
- [19]. Johansen, S., (1991). Statistical analysis for integrating vectors, *Journal of Economic Dynamic and Control*. 12: 231-254.

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