International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 1, Issue 2, October - 2013, PP 81-87
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

SINGULAR TERNARY SEMIRINGS

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Received: 08-08-2013 **Revised: 27-08-2013** **Accepted: 3-09-2013**

Abstract: *In this paper, we introduce the notions of subsemimodule generated by a subset and austere ternary S-semimodule M and observe (0: M) = (0: m) for any nonzero element m in M. Also we introduce the notions of $A_S(M)$, $T_S(M)$ and singular ideal T(S) for a ternary semiring S and obtain the characteristics of T(S). Also we observe the property of singularity is preserved under a semiisomorphism of ternary semirings.*

Keywords: *Austere ternary semimodule, Singular ideal, Semiisomorphism, Singular and non-singular ternary semirings.*

1. INTRODUCTION

It was remarked by M. Ferrero and E. R. Puczylowski in [6] "Studying properties of rings one can usually say more assuming that the considered rings are either singular or non-singular". The same remark is equally true in the case of a ternary semiring which was first introduced by T. K. Dutta and S. Kar in [1]. The notion of semiring was firstly introduced by Vandiver dated back to 1934. In 1971 Lister introduced ternary ring and regular ternary rings were studied by Vasile. To generalize the ternary rings introduced by Lister, in 2003 T. K. Dutta and S. Kar [1] introduced the notions of ternary semiring and ternary semimodule over a ternary semiring. They investigated regular ternary semiring, developed the ideal theory for ternary semirings and characterized the Jacobson radical of a ternary semiring by using ternary semimodules.

Though the notion of ternary semiring generalizes the notion of semiring but it is not merely a generalization of semiring because there are certain notions, for example, the lateral ideals which have no analogue in semirings.

In this paper, we introduce the notion of ternary subsemimodule generated by a subset and represented in terms of elements. We also introduce the notion of singular ternary semiring and characterized the singular ideals of a ternary semiring interms of essential right *k*-ideals of ternary semirings. Mainly we generalize the results of T.K. Dutta and M.L. Das [3] in semirings to ternary semirings. Our results obtained can be used to study some radical classes related to singular ideals.

2. PRELIMINARIES

In this section we collect some important definitions and results for our use in this paper.

2.1. Definition. [1] A nonempty set *S* together with a binary operation, called addition and a ternary multiplication, denoted by juxtaposition, is said to be a *ternary semiring* if *S* is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)de = a(bcd)e = ab(cde)$,
(ii) $(a + b)cd = acd + bcd$,
(iii) $a(b + c)d = abd + acd$,
(iv) $ab(c + d) = abc + abd$ for all a, b, c, d, e in S.

2.2. Definition. [1] Let S be a ternary semiring. If there exists an element 0 in S such that $0+x = x$ and $0xy = x0y = xy0 = 0$ for all x, y in S then 0 is called the *zero element* or simply the *zero* of the ternary semiring S. In this case we say that S is a *ternary semiring with zero*.

2.3. Definition. [1] Let S be a ternary semiring. If there exists an element e in S such that $eex = exe = xee = x$ for all x in S, then e is called a *unital element* of the ternary semiring.

2.4. Example. [1] Let Z_0^- be the set of all negative integers with zero. Then with the usual binary addition and ternary multiplication, Z_0^- forms a ternary semiring with zero element 0 and unital element -1.

2.5. Definition. [1] A ternary semiring S is called a *commutative ternary semiring* if $abc = bac = cba$ for all a, b, c in S.

2.6. Definition 1.6.[1] If A, B, C are three subsets of a ternary semiring S then by ABC we mean the set of all finite sums of the form $\sum_i a_i b_i c_i$ where $a_i \in A, b_i \in B, c_i \in C$.

2.7. Definition. [1] An additive semigroup T of a ternary semiring S is called a *ternary sub-semiring* if $t_1t_2t_3 \in T$ for any t_1, t_2, t_3 in T.

2.8. Definition. [1] An additive subsemigroup I of S is called a *left (right, lateral) ideal* of S if s_1s_2i(is_1s_2, s_1is_2) $\in I$ for all s_1, s_2 in S and i in I. If I is a left, right and a lateral ideal of S then I is called an *ideal* of S.

2.9. Definition. [1] An ideal I of S is called a *k- ideal* if $x + y \in I, x \in S, y \in I$ imply that $x \in I$.

2.10. Definition. [2] An additive commutative semigroup M with a zero element 0_M is called a *right ternary semimodule* over a ternary semiring S or simply a *right ternary S-semimodule* if there exists a mapping $M \times S \times S \to M$ (images to be denoted by mst for all m in M and s, t in S) satisfying the following conditions:

(i) $(m + n)st = mst + nst$,
(ii) $ms(t + u) = mst + msu$,
(iii) $m(s + t)u = msu + mtu$,
(iv) $(mst)uv = m(stu)v = ms(tuv)$,
(v). $0_Mst = 0_M = ms0_S = m0_St$ for all m, n in M and s, t, u, v in S.

In addition to the above conditions if $\sum_i me_i f_i = m$ holds for all m in M, where (e_i, f_i) is an identity element of S, then M is said to be a *unitary right ternary S- semimodule.*

2.11. Example. [2] Let $M_2(Z^-)$ be the ternary semiring of all 2×2 square matrices over Z^-, the set of all negative integers then $I_2 = \{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in Z\}$ forms a right ternary semimodule over $M_2(Z^-)$.

2.12. Definition. [2] A nonempty subset N of a right ternary S-semimodule M is said to be a ternary subsemimodule of M if

(i) $a + b \in N$,
(ii). $ast \in N$ for all a, b in N and s, t in S.

Note that the ternary subsemimodule N of a right ternary S-semimodule M always contains the zero of M. Throughout this paper S denotes a ternary semiring with zero.

3. SUBSEMIMODULE GENERATED BY A SUBSET

3.1. Definition. Let M be a right ternary S-semimodule and N be a ternary subsemimodule of M. *Then* $(N : M) = \{ x \in S \mid msx \in N$ and $mxs \in N$ for all $m \in M, s \in S \}$

$= \{ x \in S \mid MSx \subseteq N$ and $MxS \subseteq N \}$.

Note that $(N : m) = \{ x \in S \mid msx \in N$ and $mxs \in N , s \in S \} = \{ x \in S \mid mSx \subseteq N$ and $mxS \subseteq N \}$.

3.2. Remark. If S is commutative ternary semiring and N is a ternary subsemimodule of a ternary S-semimodule M then $(N : m)$ is an ideal of S.

Proof. Let $x, y \in (N : m)$. Then $msx \in N, mxs \in N$ and $msy \in N, mys \in N$ for all $s \in S$. $\Rightarrow ms(x + y) \in N$ and $m(x + y)s \in N$ for all $s \in S$. Hence $x + y \in (N : m)$. Let $s, t \in S$ and $x \in (N : m)$. Then $s, t \in S, mxs \in N$ and $msx \in N$ for all $s \in S$. Consider $m(xst)u = mx(stu) = mx(ust) = (mxu)st \in N$. Now consider $mu(xst) = (mxu)st \in N$. $\Rightarrow (N : m)$ is a right ideal of S. Since S is commutative ternary semiring, $(N : m)$ is an ideal of S.

3.3. Theorem. If N and N' are ternary subsemimodules of right ternary S-semimodule M and A, B are nonempty subsets of M then

(i) $A \subseteq B \Rightarrow (N : B) \subseteq (N : A)$,

(ii) $(N \cap N' : A) = (N : A) \cap (N' : A)$,

(iii) $(N : A) \cap (N : B) \subseteq (N : A + B)$ with equality holding if $0_M \in A \cap B$.

Proof. (i). Suppose $A \subseteq B$ and let $x \in (N : B)$. Then $bsx \in N$ and $bxs \in N$ for all $b \in B$ and $s \in S$. $\Rightarrow bsx \in N$ and $bxs \in N$ for all $b \in A$ and $s \in S$. $\Rightarrow x \in (N : A)$. Hence $(N : B) \subseteq (N : A)$.

(ii). Note that $x \in (N \cap N' : A) \Leftrightarrow asx \in N \cap N'$ and $axs \in N \cap N'$ for all $a \in A$ and $s \in S \Leftrightarrow asx \in N, axs \in N$ and $asx \in N'$ and $axs \in N'$ for all $a \in A$ and $s \in S$ $\Leftrightarrow x \in (N : A) \cap (N' : A)$.

(iii). Let $x \in (N : A) \cap (N : B)$. Then $asx \in N, axs \in N$ for all $a \in A, s \in S$ and $bsx \in N$, $bxs \in N$ for all $b \in B, s \in S$. $\Rightarrow (a + b)sx \in N$ and $(a + b)xs \in N$ for all $a + b \in A + B, s \in S$. Hence $x \in (N : A + B)$.

Suppose $0_M \in A \cap B$ and let $x \in (N : A + B)$. Then $(a + b)sx \in N, (a + b)xs \in N$ for all $a + b \in A + B, s \in S$. $\Rightarrow asx \in N, axs \in N$ for all $a \in A, s \in S$ and $bsx \in N, bxs \in N$ for all $b \in B, s \in S$. $\Rightarrow x \in (N : A) \cap (N : B)$. Hence $(N : A) \cap (N : B) = (N : A + B)$.

3.4. Definition. Let A, B be nonempty subsets of right ternary S-semimodule M. Then the ternary subsemimodule generated by A is the intersection of all ternary subsemimodules of M containing A, denoted by ASS.

3.5. Theorem. For any nonempty subsets A, B of a right ternary S-semimodule M, $ASS = \{ \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i \mid a_i, b_i \in A, s_i, t_i \in S, n, m$ are positive integers $\}$.

Proof. Let $T = \{ \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i \mid a_i, b_i \in A, s_i, t_i \in S, n, m$ are positive integers $\}$. First we prove that T is a ternary subsemimodule of M. Let $x, y \in T$. Then $x = \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i$ and $y = \sum_{i=1}^{k} c_i u_i v_i + \sum_{i=1}^{l} d_i$ for $a_i, b_i, c_i, d_i \in A, s_i, t_i, u_i, v_i \in S, k, l, n, m$ are positive integers. Then $x + y = \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{k} c_i u_i v_i + \sum_{i=1}^{m} b_i + \sum_{i=1}^{l} d_i$ and hence $x + y \in T$. Let $x \in T$ and $u, v \in S$. Then

$x = \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i$ for $a_i, b_i \in A$, $s_i, t_i \in S$, n, m are positive integers and $u, v \in S$. $\Rightarrow xuv = (\sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i)uv = \sum_{i=1}^{n} a_i s_i (t_i uv) + \sum_{i=1}^{m} b_i uv$. $\Rightarrow xuv \in T$ and hence T is a ternary subsemimodule of M. For any $a \in A$, $a = a$, $a \in T$ and hence $A \subseteq T$. To prove T is smallest, let N be a ternary subsemimodule of M containing A and let $x \in T$. Then $x = \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i$ for $a_i, b_i \in A$, $s_i, t_i \in S$, n, m are positive integers. Since $A \subseteq N$, $a_i, b_i \in N$. $\Rightarrow \sum_{i=1}^{n} a_i s_i t_i + \sum_{i=1}^{m} b_i \in N$. $\Rightarrow x \in N$. Therefore $T \subseteq N$. Hence T is the smallest ternary subsemimodule containing A.

3.6. Definition. A nonempty ternary subsemimodule N of a right ternary S-semimodule M is said to be ternary k-subsemimodule if and only if for any $m, n \in M$, $m + n \in N$ and $m \in N$ implies $n \in N$.

3.7. Definition. A ternary S-semimodule M is said to be austere if and only if $\{ 0_M \}$ and M are the only ternary k-subsemimodules of M.

3.8. Theorem. If M is austere ternary S-semimodule then $(0 : M) = (0 : m)$ for all $0_M \neq m \in M$.

Proof. Since $(0 : M) = \bigcap \{ (0 : m) \mid m \in M \}$, we have $(0 : M) \subseteq (0 : m)$ $\forall\, 0_M \neq m \in M$. Suppose if $(0 : m) \not\subset (0 : M)$ for some $0_M \neq m \in M$. Then $(0 : m) \not\subset (0 : n)$ for some $0_M \neq m \in M$, $0_M \neq n \in M$. Take $N = \{ x \in M \mid (0 : m) \subseteq (0 : x) \}$. Then $0_M \neq m \in N$ and $0_M \neq n \notin N$. $\Rightarrow \{ 0_M \} \subset N \subset M$.

Now we prove that N is a ternary k-subsemimodule of M. Let $x, y \in N$. Then $(0 : m) \subseteq (0 : x)$ and $(0 : m) \subseteq (0 : y)$. Let $z \in (0 : m)$. Then $xzs = 0$, $xsz = 0$, $yzs = 0$ and $ysz = 0$ $\forall\, s \in S$. $\Rightarrow (x + y)zs = 0$ and $(x + y)sz = 0$ $\forall\, s \in S$. $\Rightarrow z \in (0 : x + y)$.

$\Rightarrow (0 : m) \subseteq (0 : x + y)$. $\Rightarrow x + y \in N$. Let $x \in N$, $r, t \in S$. Then $(0 : m) \subseteq (0 : x)$ and $r, t \in S$. Let $z \in (0 : m)$. Then $xzs = 0$ and $xsz = 0$ $\forall\, s \in S$. $\Rightarrow xz(srt) = 0$ and $x(rts)z = 0$ $\forall\, s \in S$. $\Rightarrow (xrt)zs = 0$ and $(xrt)sz = 0$ $\forall\, s \in S$. $\Rightarrow z \in (0 : xrt)$. $\Rightarrow (0 : m) \subseteq (0 : xrt)$. $\Rightarrow xrt \in N$. Hence N is a ternary subsemimodule of M. Let $x + y \in N$ and $x \in N$. Then $(0 : m) \subseteq (0 : x + y)$ and $(0 : m) \subseteq (0 : x)$. $\Rightarrow (x + y)zs = 0$, $(x + y)sz = 0$, $xzs = 0$ and $xsz = 0$ $\forall\, s \in S$. $\Rightarrow yzs = 0$ and $ysz = 0$ $\forall\, s \in S$. $\Rightarrow z \in (0 : y)$. $\Rightarrow (0 : m) \subseteq (0 : y)$. $\Rightarrow y \in N$. Hence N is a ternary k-subsemimodule of M such that $\{ 0_M \} \subset N \subset M$, a contradiction. Hence $(0 : m) \subseteq (0 : M)$ $\forall\, 0_M \neq m \in M$. Hence the theorem.

3.9. Theorem. If I is an ideal of a ternary semiring S and M is a right ternary S-semimodule then $N = \{ m \in M \mid mIS = 0_M \text{ and } mSI = 0_M \}$ is a ternary k-subsemimodule of M.

Proof. First we prove that N is a ternary subsemimodule of M. Let $m, n \in N$. Then $mSI = 0_M$, $mIS = 0_M$ and $nSI = 0_M$, $nIS = 0_M$. $\Rightarrow (m+n)SI = 0_M$ and $(m+n)IS = 0_M$. $\Rightarrow m+n \in N$. Let $m \in N$ and $r, s \in S$. Then $mSI = 0_M$, $mIS = 0_M$. Then $(mrs)SI = m(rsS)I = 0_M$ and $(mrs)IS = m(rsI)S = 0_M$ (Since $rsS \subseteq S$ and $rsI \subseteq I$). $\Rightarrow mrs \in N$. Hence N is a ternary subsemimodule of M.

Now we prove that N is a ternary k-subsemimodule of M. Let $m+n \in N$ and $m \in N$. Then $(m+n)SI = 0_M$, $(m+n)IS = 0_M$ and $mSI = 0_M$, $mIS = 0_M$. $\Rightarrow n \in N$. Hence the theorem.

4. Singular Ternary Semirings

4.1. Definition. Let M be a ternary S-semimodule. Then for any $m \in M$, we define $A_S(M) = (0: m) = \{ x \in S \mid msx = 0 \text{ and } mxs = 0 \text{ for all } s \in S \}$.

4.2. Definition. Let M be a ternary S-semimodule. We define $T_S(M)$ as $T_S(M) = \{ m \in M \mid A_S(M)$ is an essential ideal of $S \} = \{ m \in M \mid A_S(M) \cap I \neq 0 \ \forall$ nonzero ideal I of $S \}$.

4.3. Theorem. Let M be a ternary S-semimodule. Then $T_S(M)$ is a ternary k-subsemimodule of M.

Proof. Since $A_S(0) \cap I = I \neq 0 \ \forall$ nonzero ideal I of S, $0 \in T_S(M)$. Let $m, n \in T_S(M)$. Then $A_S(m)$ and $A_S(n)$ are essential ideals of S. $\Rightarrow A_S(m) \cap A_S(n)$ is an essential ideal of S. $\Rightarrow A_S(m+n)$ is an essential ideal of S (since $(0: m) \cap (0: n) \subseteq (0: m+n)$). $\Rightarrow m+n \in T_S(M)$. Let $m \in T_S(M)$ and $r, s \in S$. Then $A_S(m)$ is an essential ideal of S. Let I be any nonzero ideal of S. Then rsI is a nonzero ideal of S. $\Rightarrow (0: m) \cap rsI \neq 0$. $\Rightarrow$ there exists $0 \neq rsx \in (0: m) \cap rsI$ where $0 \neq x \in I$. $\Rightarrow m(rsx)t = 0$ and $mt(rsx) = 0 \ \forall t \in S$. $\Rightarrow (mrs)xt = 0$ and $(mrs)tx = 0 \ \forall t \in S$. $\Rightarrow 0 \neq x \in (0: m) \cap rsI$. $\Rightarrow A_S(mrs)$ is an essential ideal of S. $\Rightarrow mrs \in T_S(M)$. Hence $T_S(M)$ is a ternary subsemimodule of M.

Let $m, m+n \in T_S(M)$ and I be a nonzero ideal of S. Then $A_S(m) \cap A_S(m+n) \cap I \neq 0$. $\Rightarrow$ there exists $0 \neq i \in I$ such that $i \in A_S(m) \cap A_S(m+n)$. $\Rightarrow msi = 0$, $mis = 0$ and $(m+n)si = 0$, $(m+n)is = 0 \ \forall s \in S$. $\Rightarrow nsi = 0$ and $nis = 0 \ \forall s \in S$. $\Rightarrow 0 \neq i \in A_S(n) = (0: n) \cap I$. $\Rightarrow A_S(n)$ is an essential ideal of S. $\Rightarrow n \in T_S(M)$. Hence $T_S(M)$ is a ternary k-subsemimodule of M.

$T_S(M)$ is called *singular ternary subsemimodule* of the right ternary S-semimodule M. The singular ideal of the right ternary S-semimodule S_S is called the (right) *singular ideal* of the ternary semiring S and is denoted by $T(S)$. i.e., $T(S) = \{ s \in S \mid s^* \cap H \neq 0$ for every nonzero right ideal H of $S \}$.

4.4. Theorem. Let S be ternary semiring. Then $T(S) = \{ x \in S \mid xIS = 0$ and $xSI = 0$ for some essential right ideal I of $S \}$.

Proof. Take $T^* = \{ x \in S \mid xIS = 0$ and $xSI = 0$ for some essential right ideal I of $S \}$. Let $x \in T(S)$. Then $x^* \cap I \neq 0$ for any nonzero right ideal I of S. $\Rightarrow xx^*S = 0$ and $xSx^* = 0$ and x^* is an essential right ideal I of S and hence $x \in T^*$.

Let $x \in T^*$. Then $xIS = 0$ and $xSI = 0$ for some essential right ideal I of S. $\Rightarrow I \subseteq x^*$. Since I is an essential right ideal of S, x^* is also an essential right ideal of S. $\Rightarrow x \in T(S)$. Hence the theorem.

4.5. Theorem. Let S be ternary semiring. Then $T(S) = \{ x \in S \mid xIS = 0$ and $xSI = 0$ for some essential right k-ideal I of $S \}$.

Proof. Take $T^{**} = \{ x \in S \mid xIS = 0$ and $xSI = 0$ for some essential right k-ideal I of $S \}$. By above theorem it is clear that $T^{**} \subseteq T^* \subseteq T(S)$. Let $x \in T(S)$. Then $x \in T^*$. $\Rightarrow xIS = 0$ and $xSI = 0$ for some essential right ideal I of S. Let $\hat{I}$ be the k-closure of I. Then $I \subseteq \hat{I}$. Since I is essential, $\hat{I}$ is also an essential right k-ideal of S. Let $a \in \hat{I}$. Then there exists $b \in I$ such that $a + b \in I$. $\Rightarrow xbs = 0$ and $xsb = 0$, $x(a+b)s = 0$ and $xs(a+b) = 0$. $\Rightarrow xas = 0$ and $xsa = 0$. $\Rightarrow x \hat{I}S = 0$ and $xS \hat{I} = 0$. $\Rightarrow x \in T^{**}$. Hence the theorem.

4.6. Definition. A ternary semiring S is said to be *singular* if $T(S)= S$ and *non-singular* if $T(S)= 0$.

4.7. Definition. A surjective morphism of ternary semirings $\gamma : S \to S'$ is called *semiisomorphism* if $ker\, \gamma = 0$.

4.8. Theorem. If $\gamma : S \to S'$ is a semiisomorphism and $T(S) = S$ then $T(S') = S'$.

Proof. Clearly $T(S') \subseteq S'$. Suppose if $T(S') \subset S'$. Then there exists $0 \neq s' \in S'$ such that $s' \notin T(S')$. Since γ is surjective, there exists $0 \neq s \in S$ such that $\gamma(s) = s'$. Since $s' \notin T(S')$, there exists a nonzero right ideal H' of S' such that $A_{S'}(s') \cap H' = 0$. Take $H = \{ x \in S \mid \gamma(x) \in H' \}$. Then it is easy to observe that H is a nonzero right ideal of S. Since $0 \neq s \in S = T(S)$, $A_S(s) \cap H \neq 0$. $\Rightarrow$ there exists $0 \neq h \in H$ such that $shS = 0$ and $sSh = 0$. $\Rightarrow \gamma(shS) = 0$ and $\gamma(sSh) = 0$. $\Rightarrow s'\gamma(h)S' = 0$ and $s'S'\gamma(h) = 0$. $\Rightarrow \gamma(h) \in A_{S'}(s') \cap H' = 0$. $\Rightarrow h \in ker\, \gamma = 0$, a contradiction. Hence $T(S') = S'$.

4.9. Theorem. If $\gamma : S \to S'$ is a semiisomorphism and $T(S') = S'$ then $T(S) = S$.

Proof. Clearly $T(S) \subseteq S$. Suppose if $T(S) \subset S$. Then there exists $0 \neq s \in S$ such that $s \notin T(S)$. $\Rightarrow 0 \neq \gamma(s) \in S' = T(S')$. Since $s \notin T(S)$. $\Rightarrow$ there exists a nonzero right ideal H of S such that $A_S(s) \cap H = 0$. $\Rightarrow \gamma(H)$ is a nonzero right ideal of S'. Since $\gamma(s) \in T(S')$, $A_{S'}(\gamma(s)) \cap \gamma(H) \neq 0$.
$\Rightarrow$ there exists $0 \neq h \in H$ such that $\gamma(h) \in A_{S'}(\gamma(s))$. $\Rightarrow \gamma(s)\gamma(h)S' = 0$ and $\gamma(s)S'\gamma(h) = 0$.
$\Rightarrow \gamma(shS) = 0$ and $\gamma(sSh) = 0$. $\Rightarrow shS, sSh \in \ker \gamma = 0$. $\Rightarrow h \in A_S(s) \cap H = 0$, a contradiction. Hence $T(S) = S$.

4.10. Theorem. If $\gamma : S \rightarrow S'$ is a semiisomorphism and $T(S') = 0$ then $T(S) = 0$.

Proof. Suppose if $T(S) \neq 0$. Then there exists $0 \neq s \in S$ such that $s \in T(S)$. $\Rightarrow \gamma(s) \neq 0$. Since $T(S') = 0$, $\gamma(s) \notin T(S')$. $\Rightarrow$ there exists a nonzero right ideal H' of S' such that $A_{S'}(\gamma(s)) \cap H' = 0$. Take $H = \{ x \in S \mid \gamma(x) \in H' \}$. Then it is easy to observe that H is a nonzero right ideal of S. Since $s \in T(S)$, $A_S(s) \cap H \neq 0$. $\Rightarrow$ there exists $0 \neq h \in H$ such that $shS = 0$ and $sSh = 0$. $\Rightarrow \gamma(shS) = 0$ and $\gamma(sSh) = 0$. $\Rightarrow \gamma(s)\gamma(h)S' = 0$ and $\gamma(s)S'\gamma(h) = 0$. $\Rightarrow \gamma(h) \in A_{S'}(\gamma(s)) \cap H' = 0$.
$\Rightarrow h \in \ker \gamma = 0$, a contradiction. Hence $T(S) = 0$.

4.11. Theorem. If $\gamma : S \rightarrow S'$ is a semiisomorphism and $T(S) = 0$ then $T(S') = 0$.

Proof. Suppose if $T(S') \neq 0$. Then there exists $0 \neq s' \in S'$ such that $s' \in T(S')$. Since γ is surjective, there exists $0 \neq s \in S$ such that $\gamma(s) = s'$. Since $T(S) = 0$, $s \notin T(S)$. $\Rightarrow$ there exists a nonzero right ideal H of S such that $A_S(s) \cap H = 0$. $\Rightarrow \gamma(H)$ is a nonzero right ideal of S'. Since $s' \in T(S')$, $A_{S'}(s') \cap \gamma(H) \neq 0$. $\Rightarrow$ there exists $0 \neq h \in H$ such that $\gamma(h) = h'$ and $s'h'S' = 0$ and $s'S'h' = 0$. $\Rightarrow \gamma(s)\gamma(h)\gamma(S) = 0$ and $\gamma(s)\gamma(S)\gamma(h) = 0$. $\Rightarrow \gamma(shS) = 0$ and $\gamma(sSh) = 0$. $\Rightarrow shS, sSh \in \ker \gamma = 0$. $\Rightarrow h \in A_S(s) \cap H = 0$, a contradiction. Hence $T(S') = 0$.

5. Conclusion

In this paper we introduced the notion of austere ternary S-semimodule M and proved that $(0: M) = (0: m)$ for any nonzero element m in M. Also we introduced the notions of $A_S(M)$, $T_S(M)$ and singular ideal $T(S)$ for a ternary semiring S and obtained the characteristics of $T(S)$. Also we observed the property of singularity was preserved under a semiisomorphism of ternary semirings. Our results obtained can be used to study some radical classes related to singular ideals.

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