International Journal of Advanced Research in Physical Science (IJARPS)
Volume 8, Issue 9, 2021, PP 11-18
ISSN No. (Online) 2349-7882
www.arcjournals.org

The superradiation phenomenon of R-N black holes can be unstable

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Abstract: *According to the traditional theory, RN black holes are stable even under superradiation conditions. However, if a total mirror is placed near RN black holes, the black holes may become unstable.Both the theory of inflation and accurate astronomical observation data show that the large-scale properties of the universe are asymptotically uneven, which is the closest approxima- tion to the theory of dS spacetime.In my research, the boundary conditions of bosons are related to cosmological constants.The conclusion is that the superradiant aperture of RN black may be unstable.*

Keywords: *boundary condition, cosmological constant, RN black holes*

1. Introduction

In 1972, Press and Teukolsky[14] proposed that it is possible to add a mirror to the outside of a black hole to make a black hole bomb (according to the current explanation, this is a scattering process involving classical mechanics and quantum mechanics[2, 8, 11, 13]).

When a bosonic wave is impinging upon a rotating black hole, the wave reflected by the event horizon will be amplified if the wave frequency ω lies in the following superradiant regime[14, 15, 17]

$$0 < \omega < m\Omega_H, \Omega_H = \frac{a}{r_+^2 + a^2}, \tag{1}$$

where m is azimuthal number of the bosonic wave mode, Ω_H is the angular velocity of black hole horizon.This amplification is superradiant scattering. Therefore, through the superradiation process, the rotational energy of the black hole can be extracted. If there is a mirror between the black hole's horizon and infinite space, the amplified wave will scatter back and forth and grow exponentially, which will cause the black hole's superradiation to become unstable.

In addition to the rotating Kerr black hole, people have also discovered that when the charged Bose field is incident on the charged R-N black hole, similar superradiation scattering may also occur. Starting from Hawking's area non- decreasing theorem, Bekenstein proved that the condition for superradiation scattering in this case is $\omega < Q/r_+$, where

q is the charge of the incident field; r+ is the radius of the black hole event horizon; Q is the black hole's charge. This

kind of superradiation scattering directly leads to the partial extraction of the Coulomb energy and electric charge of the charged black hole. Therefore, people have reason to believe that superradiation scattering may also make R-N space-time unstable. Under the background of asymptotically flat space-time, Hod[9] studied the superradiation and stability of R-N black holes under the disturbance of charged mass scalar field. He found that R-N black holes are stable even under superradiation conditions. However, if a total internal mirror is placed near the R-N Black Hole, the black hole may become unstable at this time. Both the inflation theory and precise astronomical observations show that the large-scale nature of the universe is asymptotically nonflat, the closest to the theory of space-time.

Zhu Zhiying[18] discussed the superradiation and stability of R-N DS space-time under the disturbance of charged scalar field, and got the following conclusions. The equation of motion of charged scalar field in R-N ds space-time background is obtained, and the effective potential is obtained by transforming the equation into a Schrödinger equation form. The classical theory of superradiative scattering is used to derive the conditions that the frequency of the superradiative field must satisfy. In order to study whether the superradiation can cause spatiotemporal instability, the author, on the one hand, studies the shape of the effective potential and finds that there is a potential well in the effective potential which may cause spatiotemporal instability, on the other hand, the evolution of the perturbation field with time is studied by numerical calculation. It is found that the perturbation of the charged scalar field will increase in the later period, which indicates the spatiotemporal instability. It is proved that the instability is caused by superradiation.

In recent articles[7, 12], that shows that the action of Hawking radiation under the Kerr black hole (extended to higher dimensions) can be simplified into a simpler form. We extend the conclusion to the R-N black hole, and obtain a new simple action form of Hawking radiation under the R-N black hole. In this paper, we use the conclusion that Hawking radiation has the same form as general superradiation under preset boundary conditions, and draw the action form of scalar particles entering RN black hole under preset boundary and superradiation form of general RN ds black hole. Is the same, then we conclude that because the RN ds black hole under superradiation can be unstable, then the superradiation generated by the scalar particle entering the RN black hole under the preset boundary can be unstable.

2. Description of the R-N Ds Black Hole System

The physical system we consider consists of a massive charged scalar field coupled to a Reissner-Nordström ds black hole of mass M and electric charge Q. The black-hole spacetime is described by the line element[9, 18]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

Where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}. \tag{3}$$

Here r is the Schwarzschild areal coordinate. (We use natural units in which $c = k = 1$.)

The dynamics of the charged massive scalar field Ψ in the RN DS spacetime is governed by the Klein-Gordon equation

$$[(\nabla^{\nu} - iqA^{\nu})(\nabla_{\nu} - iqA_{\nu}) - \mu^2]\Psi = 0\,, \tag{4}$$

where $A_\nu = -\delta^0 Q/r$ is the electromagnetic potential of the black hole. Here q and μ are the charge and mass of the field, respectively. [Note that q and μ stand for q/k and μ/k, respectively. Thus, they have the dimensions of

(length)$^{-1}$.] One may decompose the field as

$$\Psi_{lm}(t, r, \theta, \varphi) = e^{im\varphi}S_{lm}(\theta)R_{lm}(r)e^{-i\omega t}\,, \tag{5}$$

where ω is the conserved frequency of the mode, l is the spherical harmonic index, and m is the azimuthal harmonic index with $-l \le m \le l$.

3. The Superradiation Effect and Uncertainty Principle

We find the Klein-Gordon equation[1, 7]

$$\Phi_{;\mu}{}^{;\mu} = 0\,, \tag{6}$$

where we defined $\Phi_{;\mu} \equiv (\partial_\mu - ieA_\mu)\Phi$ and e is the charge of the scalar field.We get $A^\mu = \{A_0(x),$

0},and $eA_0(x)$can be equal to μ(where μ is the mass).

$$A_0 \to \begin{cases} 0 & \text{as } x \to -\infty \\ V & \text{as } x \to +\infty \end{cases} . \qquad (7)$$

With $\Phi = e^{-i\omega t} f(x)$, which is determined by the ordinary differential equation

$$\frac{d^2 f}{dx^2} + (\omega - eA_0)^2 f = 0 . \qquad (8)$$

We see that particles coming from $-\infty$ and scattering off the potential with reflection and transmission amplitudes

R and T respectively. With these boundary conditions, the solution to behaves asymptotically as

$$f_{in}(x) = Ie^{i\omega x} + Re^{-i\omega x}, x \to -\infty, \qquad (9)$$

$$f_{in}(x) = T e^{ikx}, x \to +\infty \qquad (10)$$

where $k = \pm(\omega - eV)$.

To define the sign of ω and k we must look at the wave's group velocity. We require $\partial\omega/\partial k >$ 0, so that they travel from the left to the right in the x–direction and we take $\omega > 0$.

The reflection coefficient and transmission coefficient depend on the specific shape of the potential A_0. However one can easily show that the Wronskian between two independent solutions, $\tilde{f}1$ and $\tilde{f}2$, of is conserved. From the equation on the other hand, if f is a solution then its complex conjugate f^* is another linearly independent solution. We find

$$W = \tilde{f}_1 \overline{\frac{d\tilde{f}_2}{dx}} - \tilde{f}_2 \overline{\frac{d\tilde{f}_1}{dx}} \qquad (11)$$

$$|R|^2 = |I|^2 - \frac{\omega - eV}{\omega} |T|^2$$

Thus,for $0 < \omega < eV$,it is possible to have superradiant amplification of the reflected current, i.e, $|R| > |I|$. There are other potentials that can be completely resolved, which can also show superradiation explicitly.

Classical superradiation effect in the space-time of a steady black hole: we know[1, 7] that $\psi = \exp(-i\omega t + im\varphi)$,and the ordinary differential equation

We see that particles coming from $-\infty$ and scattering off the potential with reflection and transmission amplitudes C and D respectively. With these boundary conditions, the solution to behaves asymptotically

$$\psi = Ae^{i\omega_H r_*} + Be^{-i\omega_H r_*}, r \to r_+$$

$$Ce^{i\omega_\infty r_*} + De^{-i\omega_\infty r_*}, r \to \infty$$

The reflection coefficient and transmission coefficient depend on the specific shape of the potential V.We show that the Wronskian

$$W \equiv \psi \frac{d\bar{\psi}}{dr_*} - \bar{\psi}\frac{d\psi}{dr_*}, \tag{14}$$

$$W(r \to r_+) = 2i\omega_H(|A|^2 - |B|^2), W(r \to \infty) = 2i\omega_\infty(|C|^2 - |D|^2), |C|^2 - |D|^2 = \frac{\omega_H}{\omega}(|A|^2 - |B|^2).$$

.Thus,for $\omega_H/\omega_\infty < 0$,it is possible to have superradiant amplification of the reflected current, i.e,if

$|A| = 0$, $|C| > |D|$. There are other potentials that can be completely resolved, which can also show superradiation explicitly.

The principle of joint uncertainty shows that the joint measurement of position and momentum is impossible, that is, the simultaneous measurement of position and momentum can only be an approximate joint measurement, and

the error follows the inequality $\Delta x \Delta p \geq 1/2$(in natural unit system).We find $|R|^2 = |I|^2 - \frac{\omega - eV}{\omega}|T|^2$,and we know

that $|R|^2 \geq -\frac{\omega - eV}{\omega}|T|^2$ is a necessary condition for the inequality $\Delta x \Delta p \geq 1/2$ to be established.We can pre-set the boundary conditions $eA_0(x) = y\omega$(which can be $\mu = y\omega$), and we see that when y is relatively large(according to the properties of the boson, y can be very large),$|R|^2 \geq -\frac{\omega - eV}{\omega}|T|^2$ may not hold.In the end,we can get $\Delta x \Delta p \geq 1/2$ may not hold.Classical superradiation effect in the space-time of a steady black hole,generalized uncertainty principle may not hold.The same goes for reverse inference[10].

4. Hawking Radiation From R-N Black Holes And Superradiation

We will prove that the Hawking radiation of the R-N black hole can be understood as the flux that offsets the gravitational anomaly. The key is that near the horizon, the scalar field theory in the space-time of a 4-dimensional R-N black hole can be simplified to a 2-dimensional field theory[7, 12].

The metric of the 4-dimensional Kerr-Newman black hole under Boyer-Lindquist coordinates (t, r, θ, φ) is written as follow (with natural unit, $\hbar = k = c = 1$)

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\phi - adt\right]^2, \tag{15}$$

where

$$\Delta \equiv r^2 - 2Mr + a^2 + Q^2 \quad , \quad \rho^2 \equiv r^2 + a^2\cos^2\theta, \tag{16}$$

a denotes the angular momentum per unit mass of certain KN black hole and Q, M denote its charge and mass. The inner and outer horizons of the Kerr-Newman black hole can be expressed as

$$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}, \tag{17}$$

and obviously

$$r_+ + r_- = 2M, \quad r_+ r_- = a^2 + Q^2. \tag{18}$$

The background electromagnetic potential is written as follow

$$A_v = \left(-\frac{Qr}{\rho^2}, 0, 0, \frac{aQr\sin^2\theta}{\rho^2} \right). \tag{19}$$

$$\Sigma = r^2 + a^2\cos^2\theta. \tag{20}$$

We assume that the preset boundary conditions under the superradiation effect are similar to the role of the ds space-time cosmological constant.The action for the scalar field in the R-N spacetime(Under the condition that the cosmological constant is not 0, the charge Q and *a* can be converted,we rewrite the action) is

$$S[\varphi] = \frac{1}{2}\int d^4x\sqrt{-g}\varphi\nabla^2\varphi$$
$$= \frac{1}{2}\int d^4x\sqrt{-g}\varphi\frac{1}{\Sigma}\left[-\left(\frac{(r^2+Q^2)^2}{\Delta} - Q^2\sin^2\theta \right)\partial_t^2 \right.$$
$$-\frac{2Q(r^2+Q^2-\Delta)}{\Delta}\partial_t\partial_\phi$$
$$\left. +\left(\frac{1}{\sin^2\theta} - \frac{Q^2}{\Delta} \right)\partial_\phi^2 + \partial_r\Delta\partial_r + \frac{1}{\sin\theta}\partial_\theta\sin\theta\partial_\theta \right]\varphi \tag{21}$$

Taking the limit r → r+ and leaving the dominant terms, we have

$$S[\varphi] = \frac{1}{2}\int d^4x\sin\theta\varphi\left[-\frac{(r_+^2+Q^2)^2}{\Delta}\partial_t^2 \right.$$
$$\left. -\frac{2Q(r_+^2+Q^2)}{\Delta}\partial_t\partial_\phi - \frac{Q^2}{\Delta}\partial_\phi^2 + \partial_r\Delta\partial_r \right]\varphi \tag{22}$$

Now we transform the coordinates to the locally non-rotating coordinate system by

$$\psi = \varphi - \Omega_{H1}t$$
$$\xi = t \tag{23}$$

Where

$$\Omega_{H1} = \frac{Q}{r_+^2 + Q^2}. \tag{24}$$

We can rewrite the action

$$S[\varphi] = \frac{Q}{2\Omega_{H1}}\int d^4x\sin\theta\varphi\left(-\frac{1}{f(r)}\partial_\xi^2 + \partial_r f(r)\partial_r \right)\varphi \tag{25}$$

We know that when sin θ = 0, the pull equation for action can conform to the above form, but the boundary becomes 0

However, if the boundary conditions are preset, the boundary conditions $\mu = y\omega$(y takes a larger number) act as sin θ, and then the effective action form of superradiation satisfies the effective action form of Hawking radiation, and it is not necessarily on the boundary of the horizon.When the boundary conditions are preset, a new path will be obtained.In the new path, we see

$$S[y,\varphi]=\frac{Q}{2\Omega_{H1}}\int d^4xye^{-y}\varphi\left(-\frac{1}{f(r)}\partial_\xi^2+\partial_r f(r)\partial_r\right)\varphi. \tag{26}$$

5. THE SUPERRADIATION PHENOMENON OF R-N BLACK HOLES MAY BE UNSTABLE

We can pre-set the boundary conditions $eA_0(x)=y\omega$(which can be $\mu=y\omega$)[3, 4, 6], and we see that when y is relatively large(according to the properties of the boson, y can be very large), $|R|^2 \geq -\frac{\omega-eV}{\omega}|T|^2$ may not hold.If the boundary conditions of the incident boson are set in advance, the two sides of the probability flow density equation are not equal because of the boundary conditions. Implying a certain probability, this also explains why the no-hair theorem is invalid in quantum effects.

The previous literature[1, 4, 5] indicates that the superradiation effect is a process of entropy subtraction. Spherical quantum solution in vacuum state[16].

In this theory, the general relativity theory's field equation is written completely.

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\frac{8\pi G}{c^4}T_{\mu\nu} \tag{27}$$

The Ricci tensor is by $T_{\mu\nu}=0$ in vacuum state.

$$R_{\mu\nu}=0 \tag{28}$$

The proper time of spherical coordinates is

$$d\tau^2=A(t,r)dt^2-\frac{1}{c^2}\left[B(t,r)dr^2+r^2d\theta^2+r^2\sin\theta d\phi^2\right] \tag{29}$$

If we use Eq(27), we obtain the Ricci-tensor equations

$$R_{tt}=-\frac{A''}{2B}+\frac{A'B'}{4B^2}-\frac{A'}{Br}+\frac{A'^2}{4AB}+\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}\dot{B}}{4AB}=0 \tag{30}$$

$$R_{rr}=\frac{A''}{2A}-\frac{A'^2}{4A^2}-\frac{A'B'}{4AB}-\frac{B'}{Br}-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^2}+\frac{\dot{B}^2}{4AB}=0, \tag{31}$$

In this time,

$=\partial/\partial r$

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^2}+\frac{rA'}{2AB}=0,$$
$$R_{\phi\phi}=R_{\theta\theta}\sin^2\theta=0, R_{tr}=-\frac{\dot{B}}{Br}=0,$$
$$R_{t\theta}=R_{t\phi}=R_{r\theta}=R_{r\phi}=R_{\theta\phi}=0 \tag{32}$$

$$\dot{B}=0 \tag{33}$$

We see that,

$$\frac{R_{tt}}{A}+\frac{R_{rr}}{B}=-\frac{1}{Br}\left(\frac{A'}{A}+\frac{B'}{B}\right)=-\frac{(AB)'}{rAB^2}=0 \tag{34}$$

Hence, we obtain this result.

$$A=\frac{1}{B} \tag{35}$$

If,

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^2}+\frac{rA'}{2AB}=-1+\left(\frac{r}{B}\right)'=0$$

If we solve the Eq,

$$\frac{r}{B} = r + C \rightarrow \frac{1}{B} = 1 + \frac{C}{r} \tag{37}$$

When r tends to infinity, and we set C=ye^{-y}, Therefore,

$$A = \frac{1}{B} = 1 - \frac{y}{r}\Sigma, \Sigma = e^{-y} \tag{38}$$

$$d\tau^2 = \left(1 - \frac{y}{r}\sum\right)dt^2 \tag{39}$$

In this time, if particles' mass are m_i, the fusion energy is e,

$$E = Mc^2 = m_1c^2 + m_2c^2 + \ldots + m_nc^2 + e \tag{40}$$

In this paper[18], the superradiation and stability of RN dS under the interference of charged scalar field are discussed, and the following conclusions are drawn. The equation of motion that the charged scalar field satisfies under the background of RN dS space and time is obtained, and it is converted into an approximate form. The conditions that the frequency of the superradiation field must be satisfied with the classical scattering theory are derived. In order to study whether this super-radiation will cause the spatiotemporal instability of RN dS, on the one hand, the author studied the shape of the effective potential and found that there is a good potential in the effective potential, which may lead to spatiotemporal instability; on the other hand, The variation of the disturbance field with time is studied numerically. It is calculated that the disturbance of the charged scalar field will increase in the later stage, which means that the instability of the spatio-temporal RN dS will be unstable. Facts have proved that this instability is caused by superradiation.

In recent article[5], if the incident boson is preset under certain boundary conditions, then the Schwarzschild black hole will produce superradiation. As a result, the original entropy of the Schwarzschild black hole will decrease. Ac- cording to traditional theory, the Schwarzschild black hole will not produce superradiation. The boundary conditions are preset. This possibility will be combined with the wave function of the coupling of the boson in the Schwarzschild black hole. The incident boson will have mirror quality, so even the Schwarzschild black hole can be the closest. I got interesting results about the possible superradiation of the Schwarzschild black hole. It seems that the boundary con- ditions of the boson equation of motion can be used as the cosmological constant of the superradiation phenomenon, but the probability is very small. The superradiation phenomenon of RN black holes may be unstable.

We know that when y is very large, the boson mass will act as a mirror outside the event horizon, and there is a potential obstacle to the effective potential. Similar to the above situation, in dS spacetime, the cosmological constant term is an obstacle to the effective potential. We know the properties of bosons. When y is larger, the possibility of obtaining it is smaller.

6. SUMMARY

According to traditional theory, that R-N black holes were stable even under superradiation conditions.However, if a total mirror is placed near an R-N black hole, then the black hole may become unstable.This suggests that the role of Hawing radiation in the Kerr black hole (extending to a higher dimension) could be reduced to a simpler form. We extend this conclusion to R-N black holes and obtain a new simple action form of Hawking radiation under R-N black holes. In this paper, we make use of the conclusion that Hawking radiation has the same form as superradiation under the preset boundary conditions, the action form of scalar particles entering RN black hole and the superradiation form of normal R-N dS black hole are obtained. On this basis, we conclude that since RN d black holes may be unstable under the action of superradiation, the superradiation produced by scalar particles entering RN black holes under the preset boundary conditions may be unstable.

7. Acknowledgements

This work is partially supported by National Natural Science Foundation of China (No. 11873025).

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Citation: *Wen-Xiang Chen (2021) "The superradiation phenomenon of R-N black holes can be unstable" International Journal of Advanced Research in Physical Science (IJARPS) 8(9), pp.11-18, 2021.*