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Tetrad in Curved Space-Time in Cosmological General Theory of Relativity

Sangwha-Yi*

Department of Math , Taejon University 300-716 , South Korea

Corresponding Author:** ***Sangwha-Yi, *Department of Math , Taejon University 300-716 , South Korea*

Abstract: *In the cosmological general theory of relativity, we define the tetrad that moves in r-axis in the curved space-time. We study an accelerated motion in curved space-time.*

Keywords: *Cosmological General Theory of Relativity; Tetrad in Curved Space-Time; Constant Accelerated Motion*

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1. Introduction

This theory's aim is to define tetrad that moves in r-axis in the curved space-time.

Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \tag{1}$$

In this case, the cosmological time t_0 is the present cosmological time for constant accelerated motion in cosmological general theory of relativity [2,3]. The ratio of the universe's expansion is

$$\Omega(t_0) = 1 \tag{1-i}$$

Hence, in this time, the cosmological general theory of relativity and the cosmological special theory of relativity do the general relativity theory and the special relativity theory.

In this time, a moving matter's acceleration is the constant acceleration a_0 in the Schwarzschild space-time.

$$a_0 = \frac{d}{dt}\left(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}\right) \tag{2}$$

$$a_0 t = \frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}} \quad , \quad u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}} \tag{3}$$

If $\frac{d\theta}{dt} = \frac{d\phi}{dt} = 0$, the solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \qquad (4)$$

In this time, if we use ψ ,

$$1 = (1-\frac{2GM}{rc^2})(\frac{dt}{d\tau})^2 - \frac{1}{c^2}\frac{1}{1-\frac{2GM}{rc^2}}(\frac{dr}{d\tau})^2$$

$$\cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau} \ , \quad \sinh\psi = \frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau} \qquad (5)$$

Therefore, r-axis's velocity V_r is

$$V_r = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} = u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}} \qquad (6)$$

According to Eq(5),Eq(6),

$$\frac{1}{c}\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} = \frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}}\sqrt{1-\frac{2GM}{rc^2}}dt \, , \quad \cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau}$$

$$= \frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}}\cosh\psi d\tau = \sinh\psi d\tau$$

$$\frac{1}{\cosh^2\psi} = 1-(\frac{\sinh\psi}{\cosh\psi})^2 = 1-(\frac{a_0 t/c}{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}})^2 = \frac{1}{1+\frac{a_0{}^2 t^2}{c^2}} \qquad (7)$$

Hence,

$$\cosh\psi = \sqrt{1+\frac{a_0{}^2 t^2}{c^2}} \quad , \quad \sinh\psi = \frac{a_0 t}{c} \qquad (8)$$

$$\cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau} = \sqrt{1+\frac{a_0{}^2 t^2}{c^2}} \, , \sinh\psi = \frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau} = \frac{a_0 t}{c} \qquad (9)$$

Therefore,

$$\frac{dt}{d\tau}=\frac{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}} \quad , \quad \frac{1}{c}\frac{dr}{d\tau}=\frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}} \tag{10}$$

2. Tetrad in Curved Space-Time

The tetrad $e_a{}^{\mu}$ is the unit vector that is each other orthographic.

$$e_a{}^{\mu}e_b{}^{\upsilon}g_{\mu\upsilon}=\eta_{ab} \tag{11}$$

Therefore, Eq(11) is

$$g_{\mu\upsilon}e_0{}^{\mu}(r,t)e_0{}^{\upsilon}(r,t)=\eta_{00}=-1$$

$$d\tau^2=-\frac{1}{c^2}g_{\mu\upsilon}dx^{\mu}dx^{\upsilon}$$

$$\rightarrow \ -1=g_{\mu\upsilon}(\frac{1}{c}\frac{dx^{\mu}}{d\tau})(\frac{1}{c}\frac{dx^{\upsilon}}{d\tau})=g_{\mu\upsilon}e_0{}^{\mu}(r,t)e_0{}^{\upsilon}(r,t) \tag{12}$$

According to Eq(10),Eq(12)

$$e_0{}^{\alpha}(r,t)=\frac{1}{c}\frac{dx^{\alpha}}{d\tau}$$

$$=(\frac{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}},\frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}},0,0) \tag{13}$$

About θ-axis's and ϕ-axis's orientation

$$g_{22}e_2{}^2(r,t)e_2{}^2(r,t)=\eta_{22}=1, \qquad e_2{}^{\alpha}(r,t)=(0,0,\frac{1}{r},0) \tag{14}$$

$$g_{33}e_3{}^3(r,t)e_3{}^3(r,t)=\eta_{33}=1, \quad e_3{}^{\alpha}(r,t)=(0,0,1/r\sin\theta,0) \tag{15}$$

And the other vector $e_1{}^{\alpha}(r,t)$ has to satisfy the tetrad condition, Eq (11)

$$g_{00}e_0{}^0(r,t)e_1{}^0(r,t)+g_{11}e_0{}^1(r,t)e_1{}^1(r,t)=\eta_{01}=0$$

$$e_1{}^{\alpha}(r,t)=(\frac{a_0 t/c}{\sqrt{1-\frac{2GM}{rc^2}}},\sqrt{1+\frac{a_0{}^2t^2}{c^2}}\sqrt{1-\frac{2GM}{rc^2}},0,0) \tag{16}$$

3. Conclusion

In the cosmological general theory of relativity, we define the tetrad that moves in r-axis in the curved space-time.

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