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Klein-Gordon Equation and Wave Function in Robertson-Walker and Schwarzschild space-time

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Abstract: *In the general relativity theory, we find Klein-Gordon wave functions in Robertson-Walker and Schwarzschild space-time. Specially, this article is that Klein-Gordon wave equations is treated by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.*

Keywords: *General relativity theory, Klein-Gordon wave equations; Klein-Gordon wave functions; Robertson-Walker space-time; Schwarzschild space-time*

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1. Introduction

In the general relativity theory, our article's aim is that we find Klein-Gordon wave equations and functions by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

The gauge fixing equation in general relativity theory

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} A^{\rho}$$

$$\rightarrow \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + \partial^{\rho}\Lambda)$$

$$= \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\sigma}\partial_{\sigma}\Lambda) \quad (1)$$

2. Klein-Gordon Wave Equation in Robertson-Walker Space-Time

Because the gauge fixing equation is the electro-magnetic wave equation, Klein-Gordon wave equation is in Robertson-Walker space-time.

The Robertson-Walker solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad (2)$$

In this time, 2-dimensional solution is

$d\Omega = 0$

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2} \quad (3)$$

The gauge fixing equation is in 2-dimensional solution[3]

$$\partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\sigma}\partial_{\sigma}\Lambda)$$

$$= \partial_\mu A^\mu + \Gamma^1{}_{10} A^0 + \Gamma^1{}_{11} A^1 + \partial_\mu g^{\mu\upsilon}\partial_\upsilon \Lambda + g^{\mu\upsilon}\partial_\mu \partial_\upsilon \Lambda + \Gamma^1{}_{10} g^{00} \frac{1}{c}\frac{\partial \Lambda}{\partial t} + \Gamma^1{}_{11} g^{11} \frac{\partial \Lambda}{\partial r} \qquad (4)$$

Hence, we can find Klein-Gordon wave equation in 2-dimentional Robertson-Walker space-time.

$$\partial_\mu g^{\mu\upsilon}\partial_\upsilon \phi + g^{\mu\upsilon}\partial_\mu \partial_\upsilon \phi + \Gamma^1{}_{10} g^{00}\frac{1}{c}\frac{\partial}{\partial t}\phi + \Gamma^1{}_{11} g^{11}\frac{\partial}{\partial r}\phi$$

$$= [\frac{-2kr}{\Omega^2(t)}\frac{\partial}{\partial r} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{1-kr^2}{\Omega^2(t)}\frac{\partial^2}{\partial r^2} - \frac{\dot{\Omega}}{c\Omega}\frac{1}{c}\frac{\partial}{\partial t} + \frac{kr}{\Omega^2(t)}\frac{\partial}{\partial r}]\phi = \frac{m^2c^4}{\hbar^2}\phi$$

$$\Gamma^1{}_{10} = \frac{\dot{\Omega}}{c\Omega} \quad , \quad \Gamma^1{}_{11} = \frac{kr}{1-kr^2} \qquad (5)$$

In this time, we can think the shape of Klein-Gordon wave function from 2-dimetional Robertson-Walker space-time. In this case, light is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2} = 0$$

$$\int \frac{dt}{\Omega(t)} = \frac{1}{c}\int \frac{dr}{\sqrt{1-kr^2}} \qquad (6)$$

Hence, matter wave function is in 2-dimetional Robertson-Walker space-time.

$\phi = A_0 \exp i\Phi$, A_0 is amplitude

$\Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \int \frac{dr}{\sqrt{1-kr^2}}$, ω_0 is angular frequency, $k_0 = |\vec{k}_0|$ is wave number

i) $k = 1, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \sin^{-1} r$

ii) $k = 0, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 r$

iii) $k = -1, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \sinh^{-1} r$ (7)

If the definition of energy and momentum is

$$E = \frac{\hbar\omega_0}{\Omega(t)}, \vec{p} = \frac{\hbar\vec{k}_0}{\Omega^2(t)} \qquad (8)$$

Energy-Momentum relation is in Robertson-Walker space-time,

$$m^2c^4 = E^2 - \frac{\Omega^2(t)}{1-kr^2}p^2c^2, E = mc^2\frac{dt}{d\tau}, \vec{p} = m\frac{d\vec{r}}{d\tau} \qquad (9)$$

Finally, angular frequency-wave number relation is in Robertson-Walker space-time,

$$\frac{\hbar^2\omega_0^2}{\Omega^2(t)} - \frac{\hbar^2 k_0^2 c^2}{\Omega^2(t)}\frac{1}{1-kr^2} = m^2c^4 \qquad (10)$$

Hence, Klein-Gordon wave equation-Eq(5) is satisfied by matter wave function-Eq(7) in Robertson-Walker space-time.

3. KLEIN-GORDON WAVE EQUATION IN SCHWARZSCHILD SPACE-TIME

Because the gauge fixing equation is the electro-magnetic wave equation, Klein-Gordon wave equation is in Schwarzschild space-time.

The Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\Omega^2] \tag{11}$$

In this time, 2-dimensional solution is

$d\Omega = 0$

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \tag{12}$$

The gauge fixing equation is in 2-dimensional solution[3]

$$\partial_\mu(A^\mu + g^{\mu\upsilon}\partial_\upsilon\Lambda) + \Gamma^\mu{}_{\mu\rho}(A^\rho + g^{\rho\rho}\partial_\rho\Lambda)$$

$$= \partial_\mu A^\mu + \Gamma^0{}_{01}A^1 + \Gamma^1{}_{11}A^1 + \partial_\mu g^{\mu\upsilon}\partial_\upsilon\Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\Lambda + \Gamma^0{}_{01}g^{11}\frac{\partial\Lambda}{\partial r} + \Gamma^1{}_{11}g^{11}\frac{\partial\Lambda}{\partial r}$$

$$= \partial_\mu A^\mu + \partial_\mu g^{\mu\upsilon}\partial_\upsilon\Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\Lambda$$

$$\Gamma^0{}_{01} = \frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}} \quad , \quad \Gamma^1{}_{11} = -\frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}} \tag{13}$$

Hence, we can find Klein-Gordon wave equation in 2-dimentional Schwarzschild space-time.

$$\partial_\mu g^{\mu\upsilon}\partial_\upsilon\phi + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\phi$$

$$= [\frac{2GM}{r^2c^2}\frac{\partial}{\partial r} - \frac{1}{1-\frac{2GM}{rc^2}}\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + (1-\frac{2GM}{rc^2})\frac{\partial^2}{\partial r^2}]\phi = \frac{m^2c^4}{\hbar^2}\phi \tag{14}$$

In this time, we can think the shape of Klein-Gordon wave function from 2-dimetional Schwarzschild space-time. In this case, light is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = 0$$

$$t = \frac{1}{c}\int\frac{dr}{1-\frac{2GM}{rc^2}} = \frac{r}{c} + \frac{2GM}{c^3}\ln|r - \frac{2GM}{c^2}| \tag{15}$$

Hence, Klein-Gordon wave function is in 2-dimetional Schwarzschild space-time-

$\phi = A_0 \exp i\Phi$, A_0 is amplitude

$$\Phi = \omega_0 t - k_0 r - k_0\frac{2GM}{c^2}\ln|r - \frac{2GM}{c^2}|$$

ω_0 is angular frequency, $k_0 = |\vec{k}_0|$ is wave number (16)

If the definition of energy and momentum is

$$E = \frac{\hbar\omega_0}{(1-\frac{2GM}{rc^2})}, \vec{p} = \hbar\vec{k}_0(1-\frac{2GM}{rc^2}) \tag{17}$$

Energy-Momentum relation is in Schwarzschild space-time,

$$m^2c^4 = (1-\frac{2GM}{rc^2})E^2 - \frac{p^2c^2}{(1-\frac{2GM}{rc^2})}, E = mc^2\frac{dt}{d\tau}, \vec{p} = m\frac{d\vec{r}}{d\tau} \tag{18}$$

Finally, angular frequency-wave number relation is in Schwarzschild space-time,

$$\frac{\hbar^2\omega_0^2}{(1-\frac{2GM}{rc^2})} - \hbar^2k_0^2(1-\frac{2GM}{rc^2}) = m^2c^4 \tag{19}$$

Hence, Klein-Gordon wave equation-Eq(14) is satisfied by matter wave function-Eq(16) in Schwarzschild space-time

4. Conclusion

We find Klein-Gordon wave equation and function in Robertson-Walker space-time. We find Kle in-Gordon wave equation and function in Schwarzschild space-time..

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