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Golden Ratio on Discrete Surface

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Abstract: *The golden ratio on the discrete surface has been discussed.*

Keywords: *Golden ratio, average hyperbolic – elliptic unit, continuous and discrete surface, pseudo pi*

1. Introduction

In the previous paper [1] the similarity between golden ratio $\phi = \frac{1+\sqrt{5}}{2} \approx 1{,}618\ldots$ and the average hyperbolic – elliptic unit being expressed on the continuous surface $s_{continuous}(1) = 2 - \frac{1}{\sqrt{1+\pi^2}} \approx 1{,}696\ldots$ was recognized.

The subject of interest of this paper is to compare the golden ratio and the average hyperbolic – elliptic unit being expressed on the discrete surface, too. Here the path is not concluded on the circumference of a circle [2] but on the average perimeter of the most favourable polygons.

2. The Pseudo Pi (π^*)

The perimeter of n-sided regular polygon $2\pi^*R$ is shorter than the circumference of a circle $2\pi R$ since pseudo pi, denoted π^*, of any n-sided polygon is smaller than π of a circle [3]:

$$\pi^* = n\sin\frac{\pi}{n} < \pi \; for \; n < \infty. \qquad (1)$$

π^* of the first three regular polygons are collected in Table 1.

Table1. *π^* of the first three regular polygons compared to π of a circle (∞-sided polygon)*

Name	Number of sides n	π^*
digon	2	2
triangle	3	$3\,sin\frac{\pi}{3} = 3\frac{\sqrt{3}}{2} = 2{,}598$
square	4	$4\sin\frac{\pi}{4} = 2\sqrt{2} = 2{,}828\ldots$
circle	∞	π

We can see that π^* rises with number of polygon sides n becoming equal to π when a polygon at $n = \infty$ converts to a circle. The increase of $\pi^*$is gradually smaller. Consequently the average π^* of two neighbour even-sided polygons is smaller than π^* of the odd-sided polygon in the middle between them. The smallest is the average pi of digon (n=2) and square (n=4) since it is smaller than π^* of triangle (n=3):

$$\overline{\pi^*_{minimal}} = \frac{\pi^*_{digon}+\pi^*_{square}}{2} = 2{,}414\ldots < \pi^*_{triangle} = 2{,}598\ldots. \qquad (2)$$

It enables the smallest and thus most favourable path concluded on the average perimeter $2\overline{\pi^*_{minimal}}R$ of the corresponding polygons on the discrete surface:

$$\pi^*_{favourable} = \overline{\pi^*_{minimal}} = \frac{\pi^*_{digon}+\pi^*_{square}}{2} = \frac{2+2\sqrt{2}}{2} = 1+\sqrt{2}. \qquad (3)$$

3. π^* and the Average Hyperbolic-Elliptic Unit

The most favourable π^*gives the next ratio of the average hyperbolic – elliptic unit $s(1)$ to elliptic unit 1 being expressed on the most favourable discrete surface:

$$\frac{s_{discrete}(1)}{1} = 2 - \frac{1}{\sqrt{1 + \pi_{favourable}^{*2}}} = 2 - \frac{1}{\sqrt{1 + (1 + \sqrt{2})^2}} = 2 - \frac{1}{\sqrt{4 + 2\sqrt{2}}} = 1{,}6173 \ldots \quad (4)$$

4. The Average Hyperbolic-elliptic Unit Being Expressed on the Discrete Surface Compared to the Golden Ratio

The average hyperbolic-elliptic unit being expressed on the most favourable average discrete surface $s_{discrete}(1) = 2 - \frac{1}{\sqrt{4+2\sqrt{2}}} = 1{,}6173 \ldots$ only on the fourth decimal differs from the golden ratio $\phi = \frac{1+\sqrt{5}}{2} = 1{,}6180 \ldots$

Since:

$$\phi - s_{discrete}(1) = 1{,}6180 - 1{,}6173 = 0{,}0007. \quad (5)$$

5. The Average Hyperbolic-elliptic Unit Being Expressed on the Discrete as well as Continuous Surface Compared to the Golden Ratio

$$s_{discrete}(1) = 1{,}6173 < \phi = 1{,}6180 < s_{continuous}(1) = 1{,}6967. \quad (6)$$

The golden ratio lies within the interval defined by the discrete and the continuous unit.

6. Conclusion

The golden ratio almost equals the average hyperbolic – elliptic unit if the latter is expressed on the most favourable average discrete surface which is characterized by the value of pseudo pi yielding $\pi_{favourable}^{*} = 1 + \sqrt{2}$.

Dedication

To Mahatma Gandhi and his quote: “True beauty after all consists in purity of heart.”

Figure1. *About true beauty [4]*

References

[1] Janez Špringer (2022) “Golden Background of Beauty” International Journal of Advanced Research in Physical Science (IJARPS) 9(12), pp.9-11, 2022.

[2] Špringer J. Fine Structure Constant as a Mirror of Sphere Geometry. Progress in Physics, 2013, v. 1, 12–14.

[3] https://www.mathsisfun.com/geometry/regular-polygons.html

[4] https://kwize.com/quote/3336

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