Statistical Distribution Functions for Daffodil Plants – Directions & Numbers

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Abstract: N = 193 N. pseudo-Narcissus plants (daffodils) are statistically accounted for, in terms of the number of flowering stems per plant and also the preferred North, East, South, or West phototropic direction.

An exponential distribution function of the form f(x)dx = A exp (-ax) dx accurately determines the distribution of group size, correlation r = 0.99, with integration constants A = 0.12, a = 0.1.

Re-ordering compass point directions in a clockwise sequence East, South, West, then North it is determined that an exponential function based on polar angle predicts the preferred directions of the daffodil plants. This allows statistical interpolation for intermediary compass points, i.e. Southeast, Southwest, and Northwest. The directional distribution function is h (y) dy = A exp (-a y) dy, A = 1.6, a = 0.7, r = 0.96.

Keywords: *N. pseudo-Narcissus*, daffodil plant, Alzheimer drug development, alkaloid compounds, compass point orientation, phototropism, Poisson function, exponential distribution

1. INTRODUCTION

The daffodil plant (*N. pseudo-Narcissus*) typically makes its first appearance during March of each year, as early as March 1st following a warm winter, (Hanks, 2003; Greene et al. 2019). (Of course, these initial dates may occur 1 or 2 weeks later just 200 miles North to Boston, or 1 or 2 weeks earlier 200 miles South to Washington D.C.) Similar to the photo-sensitive responsibilities (also called heliotropic or phototropic responses) of sunflower plants *(the Helianthus,* Vandenbrink et al., 2014), daffodil plants also align and position their flower stems during development, with respect to sunrise, noontime, and sunset, so as to collect maximum solar radiation averaged throughout the day. The development phase, from ground level break-through to the appearance of flowers, usually takes 3 to 5 weeks, after that, the flowers last 3 or 4 weeks and then wilt. Remarkably, as shown in Fig. 1 below, for each plant, most of the flowers seem to "agree", concurring on the optimal direction, which is usually Eastward facing, but can sometimes be South, West, or North (listed in descending preferred order).

Figure1. *Normal growth of the daffodil plant, 3rd week of April*

Introduction cont

In terms of plant mechanics, elastic modulus, stem bending, and buckling characteristics are measured by Greene & Greene (2015, 2016, 2017, 2019) for the *Taraxacumofficinale* and *N. pseudo-Narcissus*

plants. These intrinsic plant quantities are of interest, because *alkaloid* extracts, distilled and refined from daffodil stems, are used as the chemical basis for anti-Alzheimer drugs.

Daffodils can propagate as a result of pollination by bees, wasps, or small birds, whereby pollen from a male flower is mixed with pollen from a female flower, and then deposited elsewhere. However, in modern times, for the most part, propagation is accomplished by replanting bulbs during the Fall or Winter months, which subsequently spout in the Spring, because the daffodil plants require a long hibernation period over the Winter months to prepare. Plant growth characteristics, in particular their response to sunlight, are discussed by Gates et al. (1965), McKim (2019), Liu et al. (2006), Waring 1983, and Kutschera & Niklas (2013).

2. MATERIALS AND METHODS

Statistical data on daffodil plants and their preferential directions are collected for this study during March and April over a 4 square mile area in South Huntington N.Y., latitude ~ 41^0 N., along East-West and North-South roads, extending 1- mile in each direction. Details are provided by Greene & Greene (2019). A total of N=193 daffodils are recorded in terms of # number of flower stems per plant, and their preferred North-East-South-West orientation direction. These tabular data are presented in graphical format in Figures 2 and 3 below.

In order to determine the statistical distribution function for plant size, the *N. pseudo-Narcissus* plants (daffodils) are organized into 5 groups, those with 1- to- 5, 6- to- 10, 11-to- 15, 16- to- 20, or 21-to- 25 flowering stems per plant, as shown in Figure 2 below.
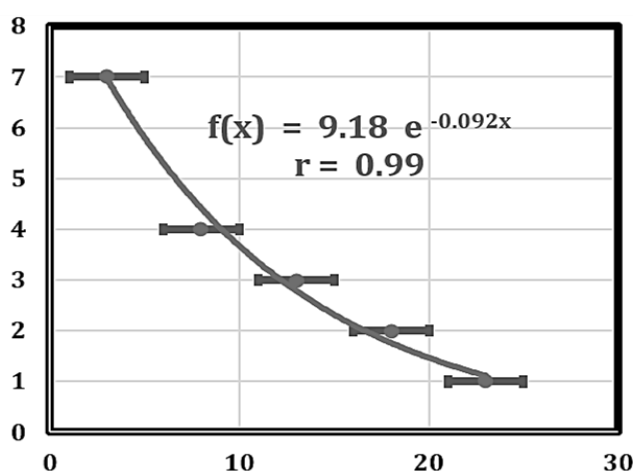

Figure2. *Exponential distribution for plant size, in terms of # of flowering stems per plant. Error bars indicate the statistical compartments or sorting "bins", at intervals of 5 flower stems per bin, with each sorting bin corresponding to 1 to 5, 6 to 10, 11 to 15, 16 to 20, or 21 to 25 flower stems per plant.*

2.1. Optimal Distribution Functions

An investigation of linear regression, logarithmic, and power law functions determines that an ordinary first-order exponential distribution does an excellent job (correlation r = 0.99) at predicting the number of flowers per plant:

Eq. (1) $\quad$ **f (x) dx = A exp (-a x) dx**

This function is normalized by integrating from x = 1 to x = 25, so that the total integrated probability is Pr = 1.0 over the experimental range:

X = 25

Eq. (2) 1.0 $\quad$ = ∫A exp (-a x) dx $\quad$ = [(A / a) exp (-a x)]

x = 1

2.2. The Preferred Direction Distribution

In terms of the North, East, South West distribution function, Figure 3 below is re-sequenced in descending order of popularity East, South, West then North as shown below:
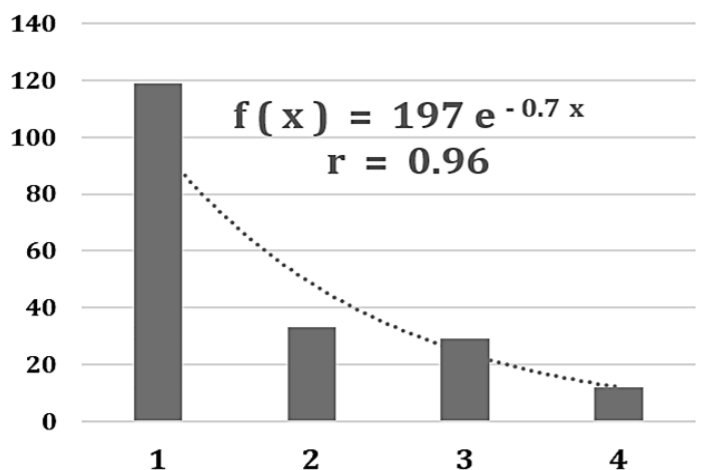
Figure3. *Exponential distribution function for preferred daffodil compass directions*

Where 1 = East, 2 = South, 3 = West, 4 = North.

This numbering scheme, while somewhat difficult to work with, allows continuous interpolation for the expected flower distribution patterns, including the off-axis compass points, whereby Southeast = 1.5, Southwest = 2.5, and Northwest = 3.5. At present, we have not determined how to account properly for the North-East direction, lacking exact plant orientation data on this off-axis direction.

3. RESULTS

After normalizing these distribution functions, to determine the integration constant, the integral over the experimental range is given by Pr = ∫= 1.0, with integration constants given by :

Eq. 3 **f(x) dx = 0.121 exp (- 0.1 x) dx** for #'s per plant, applicable from x = 1 to x = 25, and

Eq. 4 **h(y) dy = 1.6 exp [-0.7 y] dy** for plant direction, useful from y = 1 (East) to y = 4 (North)

Appendix I discusses additional mathematical procedures, in terms of evaluating these integrals for 5 practical examples.

4. DISCUSSION

An exponential distribution function, similar to the Poisson distribution (k=1), is found to accurately predict the # of flowers per plant, with excellent correlation r = 0.99 [f (x) dx = 0.12 exp (- 0.1 x) dx]. In addition, the first-order exponential function also predicts the distribution of flower directions, North-East-South-West, with correlation coefficient r = 0.96 [h(y) dy = 1.6 exp (-0.7 x) dy].

4.1. Cross Correlations

In the future, the flower number distribution function f(x) and the flower direction distribution h(y) can be combined, to form a multi-variable "*cross correlation" function*, of the form g(x,y) = f(x)* h(y), so that crop yield estimates can be made (in terms of tons per acre, or metric tons per hectare) :

∬ g(x,y) dx dy = ∫ ∫ f(x) h(y) dx dy

However, at our current stage of knowledge, we simply do not have enough experimental evidence that the # number of flowers per plant and the resulting compass directions of the flowers are partially correlated or statistically independent.

4.2. North-East-South-West Preferred Directions

Experimentally, we have measured N=193 flower stems, with the result that approximately 65% of the flowers "decide" to face East, 15% face South, 14% face West, and only 6% face North. In order to make use of the exponential distribution function h(y), it is necessary to re-order these directions, in the sequence East, South, West, then North, assigning the numbers 1 = East, 2 = South , 3 = West, and 4 = North, as shown in Fig. 3. This technique has the advantage of being able to predict the off-axis directions SouthEast (y=1.5), Southwest (y=2.5), and Northwest (y=3.5), in addition to a continuous distribution of all compass directions.

However, one limitation of these results so far, is that we cannot predict the North-East direction (lacking experimental data), where the percentage suddenly jumps from a minimum of just 6% (facing North) to a maximum of 65% (facing East). Instead of using an exponential, ultimately, some sort of modified Sine or Cosine function may prove more useful, in terms of predicting the preferred compass directions. An exponential extrapolation of the current data (Fig. 3) indicates that only 3% of the flower stems are expected to face Northeast, but this may be an under-estimate.

4.3. Practical Applications

The main application of this research, in terms of studying the growth characteristics of the *Narcissus* plant (daffodil) is the development of modern anti-Alzheimer drugs, which are derived by refining *alkaloid* extracts from the *Narcissus* stems. (The *alkaloids* are extremely powerful chemical compounds, with 100's of different types of variants, depending on the plant and location.) Since the *Narcissus* stems and flowers are very sensitive to sunlight, it is likely that each plant's biochemical factors vary in strength, according to accumulated solar exposure during the critical early weeks of development.

4.4. Commercial Property Yield Factors

For instance, according to the statistical results presented here, after planting 100 daffodil bulbs, one might expect an average of 7 to 10 flowers per plant, with 60 to 65% of these flowers face East (to accumulate maximum exposure to sunlight). Commercially, since a definite annual crop yield is required for a particular application (measured in tons per acre), it is quite possible that just 1-acre, or 10-acres, or even as many as 100-acres is required, to obtain the necessary tonnage. So it is important to mathematically understand the sub-set distribution patterns, to appreciate and plan exactly which plants are being generating on a year-by-year basis.

4.4.1. Logic of the exponential function

A priori, at the present time, we have no fundamental explanation, in terms of a basic reason, in terms of why the first-order exponential distribution (the Poisson function, k = 1) does such an excellent job (r = 0.99 and r = 0.96) at predicting both the # number of flowers per plant, and the preferred compass directions of the daffodil flowers. Possibly, the age of the individual plants may be a relevant factor, because on a year-by-year basis, with accumulated exposure, re-adjusting to the environment, the # number of flowers generated by each plant increases, i.e. the plants get larger and larger. Our personal experience*(unpublished)* indicates that during the first year after planting a bulb, only 2, 3, or 4 flowers may result, but in subsequent years, this may expand to 8 or 10 or more. The range of possible values include a minimum of N=0 flowers for a new plant, up to a maximum of N=25 or more for a mature plant. One limitation of the present study, is that we did not collect data for the N=0 case (i.e. no flower stems), which conceivably might change the statistical distribution patterns.

4.4.2. Comparison with other studies

McMahon (1974), Costes et al. (2008), and Durand et al. (2005) present various statistical and biomechanical models for plants and trees, in terms of the structural statistics of branching patterns, a concept which is similar to the stem development patterns of flowering plants, such as the daffodil as discussed here. Bernoulli and Markov mathematical models are employed by these authors with some success, but as these distributions can be quite complicated, a first-order exponential distribution (i.e. the k=1 Poisson) does an excellent job, as presented here, both in terms of the # number of flowers per plant, and the preferred flower-stem directional orientation along the primary North-East-South-West compass points.

4.4.3. Appendix I Evaluating the Distribution Functions

The exponential distribution function is an almost perfect (correlation coefficient r = 0.99) approximation to the experimental data, in terms of the number of flowers per plant (see Fig. 2):

f (x) dx = A exp (-a x)dx = 0.12 exp (- 0.1 x) dx   [1]

where   A = 0.12   and   a = 0.1

Example 1 - As an example of how such a function is used, as a practical matter, say that you want to calculate the % percentage of plants that fall in the range of 1 to 10 flowers per plant. This then is simply a matter of evaluating the integral over the limits x = 1 to x = 10:

$X = 10 = \int$ A exp (-a x) dx   = [(A / a) exp (-a x)]   (2)

X = 1 = (0.12 / 0.1) [0.905 - 0.368]   = 0.65 = 65 % percent

Example2- As a further example, the limits are expanded to integrate the distribution from x = 1 to x = 20 flowers yielding:

(0.12 / 0.1) [0.905 – 0.135] =   0.93 = 93 % percent

In words, this simply means that 93% percent of the daffodil plants will yield from 1 to 20 flowers per plant.

Example3- Preferred directions.

In order to normalize the directional distribution function, it is necessary to evaluate h (y) it over the range y = 1 to y = 4 :

1.0 = (A / a) $\int$exp (- a y) dy   which integrates to :

0.7 / A =   [exp (-0.7 (4)) - exp (-0.7 (1))]

Yielding the results A =   1.6 and   a = 0.7. So, the directional distribution function h (y) is given by:

h(y) dy = 1.6 exp [-0.7 y] dy

Example 4 - In terms of integrating the directional distribution function h (y) dy,   say you want to calculate the % percent of expected flower stems (probability = Pr) from due South (y = 2) to due West (y = 3):

Pr =   (1.6 / 0.7) [exp (-0.7 (2)) - exp (-0.7 (3))]

                =   2.3 [0.25 - 0.12] = 0.30 = 30 %

4.5. Error Estimates and Tolerance

While r = 0.99 is an excellent correlation, rarely achieved doing work of this type, it is nevertheless not "perfect". For instance, with a sample size of N ~ 200, as used here, an error will accumulate (because the exponential function is not 100% perfect) so that when integrating f (x) dx over the desired range, an error of +/- 1.0 flower stem, i.e. an error of +/- 0.5 % in terms of the # of stems per plant accumulates from the distribution integral f(x) dx .

In terms of the directional distribution function h (y) dy, with correlation r = 0.96, again with N ~ 200 stems, integrating over the desired range of compass points, an error of +/- 2.0 flower stems may accumulate, i.e. an error of +/- 1 %, still quite acceptable.

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